Debt Overhang and Inefficient Capital Reallocation

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Abstract

This paper develops a general equilibrium model of firm over-borrowing with debt overhang, providing a novel angle for evaluating preventive policies during a corporate credit boom. When capital decisions must be made under idiosyncratic uncertainties, firms’ individually optimal investments ex-ante fail to internalize their effects on raising equilibrium asset price and overall indebtedness which, through debt overhang, squeezes borrowing capacity ex-post, weakens reallocation and depresses productivity. This pecuniary externality leads to a wedge between private and social costs of debt, thereby leaving room for regulations. Optimally set ex-ante interventions resolve the time consistency problem associated with ex-post stimulus, whereas restrictions on macroprudential measure create a role for commitment. Subsidizing new debts reduces ex-ante incentive to borrow and thus is more desirable than bailing out existing ones.

We quantify the magnitude of inefficiency using an augmented industry dynamics model parameterized by a pan-European firm-level dataset. In the model, capital reallocation efficiency drops by 20% less in credit crunch if the economy ex-ante de-leverages to the constrained efficient level. In addition, the excessive accumulation of long-term debt during years of credit boom leads to declining capital reallocation efficiency not only in boom years but also in the credit bust that may follow.

Keywords: Pecuniary externality, Debt Overhang, Capital Reallocation, Inefficiency, Macroprudential Policy

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1 Introduction

Leverage of the non-financial corporate sector is higher today than it was before the Great Recession. From 2010 to 2016, US corporate debt grew by 37%, whereas real GDP increased by around 12%. Amid the corporate debt boom are increasingly heightened discussions on whether policy measures should be implemented. The September 2016 Federal Open Market Committee (FOMC) meeting minutes reads: “...a few participants expressed concern that the protracted period of very low interest rates might be encouraging excessive borrowing in the non-financial corporate sector...”; more recently, France’s Central Bank said that it would increase banks’ capital requirements in order to curb the borrowing binge.\footnote{US data from Federal Reserve; FOMC link; France link; Earlier contributions on this question includes Gromb and Vayanos (2002), Lorenzoni (2008), Jeanne and Korinek (2010), Farhi and Werning (2012), Stein (2012), Jeanne and Korinek (2013), He and Kondor (2016), Korinek and Simsek (2016) and Dávila and Korinek (2018). See Korinek and Mendoza (2014) for an overview of related studies.} The de-leveraging policy proposal raises a natural question: why and under what condition is corporate credit boom socially inefficient?\footnote{If firm’s private incentive to incur debt and their heavy reliance on borrowing is aligned with that of the society, then policy implementations that intend to reduce borrowing would be less called-for. With corporate indebtedness reaches to new historical level, this issue becomes particularly pressing.} If firm’s private incentive to incur debt and their heavy reliance on borrowing is aligned with that of the society, then policy implementations that intend to reduce borrowing would be less called-for. With corporate indebtedness reaches to new historical level, this issue becomes particularly pressing.

In this paper, we offer a novel ex-ante view in accessing the need for preventative policy in the presence of a corporate credit boom. We show firms’ individually optimal borrowing and investment decisions leads to excessive debt overhang, which results in socially inefficient capital reallocation that depresses aggregate productivity. In the second-best point of view, firms over-borrows. Limiting the amount a firm can borrow, either through quantity-based policies, such as debt cap, or price-based policies, such as proportional taxes on borrowing, lead to more efficient usage of the economy’s capital.

In our model, firms invest ex-ante for production, but they face uncertainty regarding their idiosyncratic productivity. Once uncertainty is realized, installed capital at the hand of each firm can be reallocated through a competitive market. As both investment and capital reallocation are financed with debt, a debt-overhang situation arises: the existing debt, which finances investment ex-ante, is senior to new debt, which finances capital reallocation ex-post. Because total debt capacity is limited, a firm that needs further external financing when it becomes productive is constrained by the amount of debt incurred when it invests. As firms invest ex-ante, they fail to internalize the fact that their investment, as a whole, raises capital price, making every firm more indebted, squeezing the debt capacity for all potentially productive firms ex-post, hindering capital reallocation and depressing aggregate productivity of the economy. To the society, the decreasing aggregate efficiency through rising asset price adds extra cost
borne by everyone. In other words, with the capital price determined in general equilibrium, the presence of debt overhang lead to a wedge between private and social costs of debt.

The key element that gives rise to debt overhang is that firms have limited commitment, thereby they can only pledge a limited fraction of its future cash flow to the outside investors. At the social level, this induces a negative relationship between the debt that finances investment and the debt that finances capital reallocation, i.e., a trade-off between the total amount of capital used for production and how efficient it is. At the competitive equilibrium, a regulatory reduction in firm’s ex-ante leverage leads to a drop in investment demand as well as capital price, implying firms incur less debt for a given amount of investment. With less prior debt commitment hanging over them, ex-post productive firms’ debt capacity is relaxed and they can finance more capital purchase, leading to an improved reallocation efficiency. At the social level, the benefit of improvement in ex-post reallocation efficiency exceeds the cost of reducing ex-ante investment. However, as the gains from de-leveraging come from capital price, it does not happen in the competitive equilibrium as firms are atomic price takers.

Our model highlights a particular social cost in credit-driven asset price boom which weakens productivity-enhancing reallocation. Based on the inefficiency result, we derive two formulas that characterize the optimal ex-ante corrective policy. On the one hand, the government could impose a simple loan-to-value (LTV) limit to reduce borrowing; On the other hand, it also could raise the cost of debt through tax. As the externality originates from increasing in capital price, we show that the magnitude of the policy proposal should increase for capital types whose supply schedule is less elastic, such as land.

The forward-looking nature of asset price in our model implies whether ex-post interventions are effective or not depend on how this intervention would affect firm’s ex-ante incentive. We consider two commonly adopted ex-post policies: a subsidy on the debt accumulated ex ante, or “debt bailout,” and a subsidy on new borrowing, “monetary stimulus.” We show that the two category of ex-post intervention provide opposite incentives for corporate borrowing ex-ante. Bailing out ex-ante debt increase the debt capacity for the productive firms and improves reallocation efficiency, it nevertheless raises the ex-ante incentive to borrow. As a result, asset price ex-ante increase to an even higher level than the competitive equilibrium. From an ex-ante point of view, debt bailing out induce a moral hazard problem that makes it counter-productive. On the contrary, the subsidy of new borrowing ex-post reduces the ex-ante incentive to borrow because as firms borrow ex-ante, they will get less subsidy due to debt overhang. We show that

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3For example, in the 2009 Auto Industry Bailout, the U.S. Department of the Treasury invested $80.7 billion in the big three automakers: General Motors (GM), Chrysler and General Motors Acceptance Corporation (GMAC), who warned the Congress that, without a bailout, they will face bankruptcy, causing a loss of jobs about one million (source: U.S. Department of the Treasury); In the case of Japan in 1990s, Caballero, Hoshi, and Kashyap (2008) mentioned almost one-half of public funds injected to the banking system had been used for bailing out existing loans in various ways: granting new loans, interest concessions, debt forgiveness and payment moratorium, etc.
subsidizing of new borrowing, once properly implemented, can reduce ex-ante asset price and restore constrained efficiency.

As policy makers in practice access to a wide range of ex-ante and ex-post policies, we study the optimal mix of interventions in respond to systemic crises. The common narrative points out the time consistency issue in discretionary use of ex-post interventions: the planner will find it optimal to revise its ex-ante announced plan when crisis indeed occurs. Policy implications arise from our model runs against this view. In particular, we show that if macroprudential policy can be set optimally, it effectively corrects the ex-ante borrowing incentive of private agents in response towards ex-post interventions, despite asset price being forward-looking. As a result, optimal policy mix under discretion agree with the one under commitment, and there is no time inconsistency. Nevertheless, if macroprudential interventions are restricted so that it cannot reach its optimal level—for example, due to circumvention or insufficient legal mandate—time consistency issue arises, thereby creating room for ex-ante commitment. Comparing with discretionary management, planner that commits will engage more subsidy on new debt (monetary stimulus) and less subsidy on old debt (bailing out), because the former effectively reduce ex-ante incentive to borrow due to the presence of debt overhang, while the latter provides opposite incentive.

Our quantitative exercise is based on a comprehensive pan-European private firm dataset ORBIS-Amadeus (2004-2016), and we focus on manufacturing firms. Our purpose here is to provide a gauge of the magnitude of the social cost of debt overhang. To this end, we calibrate an industry dynamics model in the spirit of Hopenhayn (1992).\(^4\) In this model, ex-ante identical firms enters the economy by making start-up investment financed by a geometrically decaying long-term loan. As firms enter, their idiosyncratic productivity is realized, and they start to finance capital growth through rolling over short-term loans. For computational tractability, firms are only allowed to issue long-term debt as they enter the market. We show the model can capture the maturity structure of corporate loans, together with a set of standard firm dynamics moments such as size distribution and firm employment, capital, output growth.

The incorporation of a senior long-term debt issued at entry give rise to debt overhang. Consistent with the static setting, un-internalized peculiar externality increase asset price, driving a wedge between the private and social cost of long-term debt. The presence of debt overhang induced externality introduces two margins that determine the counterfactual predictions of the quantitative model. First, high indebtedness means it becomes costlier to serve the debt, as the firms need to make more coupon payment; Second, overhanging debt commitments constrains the short-term capacity; As we show, the two mechanisms act like “taxes” for the young and pro-

\(^4\)In order to focus on the peculiar externality here, we let risk-free rate and wage rate to be exogenously given. Otherwise, one cohorts’ capital decision might spill over on other cohorts as risk-free rate and wage rate moves endogenously – another type of externality that is standard in the overlapping generation models (Blanchard and Fisher, 1989).
ductive, because they are more likely to be constrained by external financing. At steady state, the model predicts increase in measured industrial productivity if the long-term-debt issuance is regulated; Although the number is modest at steady state, the model predicts sizable gains regarding capital reallocation efficiency in response to a one-time unexpected credit crunch. In particular, we show implementing the constrained planner’s allocation ex-ante results in 20% smaller in MPK dispersion during the credit crunch.

One particularly puzzling phenomenon regarding the Southern European sovereign is the fact that the dispersion of return to capital across firms increases not only in credit booms periods, in which interest rate is low, but also for credit bust, in which the interest rate suddenly rises. Our quantitative exercise also shed light on this fact. Low interest rate in credit boom years crowd the less productive into operation \cite{Reis2013, Reis2015}, and it also makes firms accumulate too much long-term debt. As interest rate suddenly rise, it crowds out those less efficient firms, a cleansing force. However, it also makes it difficult for operating firms to roll over their short-term debt. The latter becomes particularly sever when there is too much ex-ante debt hanging over. It makes young and productive firms grow asymmetrically slower, as external financing is more valuable for them. It, in turn, creates a sulllying force. We show that in our calibrated economy, the latter mechanism dominates for the short run, i.e., about two years after interest rate jumps, in which capital reallocation efficiency continues to decline. In this sense, the over-accumulation of debt in boom periods also implies persistently worse capital reallocation in credit bust.

Exploiting the fact that 99% of firms in the ORBIS-Amadeus sample are bank-dependent borrowers, we provide a set of stylized facts that link the presence of debt overhang the related efficiency of capital reallocation. We first document that long-term debt accounts for the majority of bank debt: about 50% of total debt on average and about 60% to 70% for young firms with ages smaller or equal to 10. Second, at a narrowly defined sector level, higher long-term debt holding before the European Debt Crisis is associated with larger and more persistent increase in MPK dispersion after the crisis; Third, at the firm level, we document that debt accumulation dampen the positive relationship between firm’s capital growth and productivity growth. This dampening effect seems to be more pronounced for debt with long maturities; however, as our dataset does not contain direct and plausibly exogenous variations in debt overhang that enables us to study the causal role of overhang in affecting capital reallocation, we think our empirical exercise here as suggestive.

**Related Literature.** Our paper is related to several strands of literature. First, it adds to a growing literature on pecuniary externalities in incomplete markets. Geanakoplos and Polemarchakis \cite{Geanakoplos1985} established that when markets are incomplete, the competitive equilibrium may be (constrained) inefficient. The literature mainly focuses on the mechanism of fire sale externality induced by a collateral constraint \cite{Shleifer1992}. The key assumption
in this strand of literature is that firms need to dismantle a certain fraction of capital in order to be in operation. As firms invest ex-ante, they do not internalize the fact that their investment may make the price of the collateral “too low” in recessions in which the collateral constraint binds. The downward pressure in asset price tightens the borrowing constraint, amplifies the aggregate shock ([Gromb and Vayanos, 2002; Caballero and Krishnamurthy, 2003; Lorenzoni, 2008; Jeanne and Korinek, 2010, 2013; Bianchi, 2010, 2011; Stein, 2012]). While the sign of pecuniary externality is unambiguously positive in these papers, Gersbach and Rochet (2012), He and Kondor (2016) build models to show externality can change sign with the state of the economy, allowing for two-sided inefficiencies. The novelty our model is the interplay between ex-ante and ex-post borrowing induced by debt overhang. The competitive equilibrium that we describe do not have fire sale as ex-post asset price is insensitive to aggregate holding of entrepreneurial asset. Our emphasis is the general equilibrium force ex-ante, where firm investment result in co-movement between asset price and leverage which, through debt overhang, leads to real cost of the economy by weakening the subsequent asset reallocation.

The explicit modelling of productivity heterogeneity within the non-financial corporate sector means macroprudentical policy have additional implications. First, the role of optimally set ex-ante de-leveraging policy could prevent productivity in downturn from being “too low”. As the amount of investment is fixed ex-ante, the real effect of macroprudential policy proposal is to improve asset allocation entrepreneurs instead of preventing excessive liquidation/fire-sale of productive assets. Second, the fact that entrepreneurs have different productivity ex-post implies injections of public funds have larger effect if they are received by entrepreneurs with binding financial constraint. We show that, under some specification, it is optimal for the planner ex-post to engage in subsidizing new debt only: bailing out existing debt does not put public fund to best use. In this regard, our model can be seen as an attempt towards one of the research directions pointed out by Bianchi and Mendoza (2018).

Our paper is also related to the literature on debt overhang. Myers (1977) points out that outstanding debt may distort the firm’s investment incentives ([Leland, 1994, 1998; Titman and Tsyplakov, 2007; Diamond and He, 2014]). While the literature typically provides analysis of the cost for debt overhang in distorting a single firm’s investment, a few papers show that the presence of debt overhang could distort investment decision at the social level ([Lamont, 1995; Philippon, 2009; Philippon and Schnabl, 2013; Gomes, Jermann, and Schmid, 2016; Chen and Manso, 2017]). In these models, the focus is on the interaction of the household and other sectors of the economy (for example, banking), and outstanding debt claims that lead to the subsequent overhang is specified as given. Our model suggests that agents’ endogenous ex-ante debt decision may lead to constrained inefficient asset allocation in general equilibrium. Debt overhang has social cost because it depresses reallocation and productivity.

Our paper is related with the literature on capital allocation/reallocation ([Barlevy, 2002;
Eisfeldt and Rampini, 2006; Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Buera, Kaboski, and Shin, 2011; Song, Storesletten, and Zilibotti, 2011; Cui, 2013; Midrigan and Xu, 2014; Moll, 2014; Dong, Wang, and Wen, 2016; Ai, Li, and Yang, 2016; Liu, Wang, and Xu, 2017; Bleck and Liu, 2018; Lanteri, 2018). Recent empirical studies document that recessions in general, and the 08 financial crisis in particular, are followed by persistent slowdown in productivity-enhancing reallocation (Kehrig, 2015; Foster, Grim, and Haltiwanger, 2016). Theoretical contribution attribute this puzzling facts to pro-cyclical credit condition (Barlevy, 2002; Cui, 2013), (endogeneous) non-convex capital adjustment costs (Eisfeldt and Rampini, 2006; Lanteri, 2018), and search friction (Dong, Wang, and Wen, 2016). The collateral constraint augmented with debt overhang in our model highlight that debt incurred during boom years could endogenously constrain the expansion of productive firms in recession, worsening equilibrium capital reallocation. The workings of the model, i.e. debt overhang, seems to be consistent with the prediction of recent empirical work in Blattner, Farinha, and Rebelo (2018a), who find negative causal relationship between firm level indebtedness and the ability to take up profitable investment opportunities, and this relationship is mostly concentrated on firms that are heavily leveraged.

Our empirical exercise is related to the study of resource reallocation in the context of the European area (Reis, 2013, 2015; García-Santana, Moral-Benito, Pijoan-Mas, and Ramos, 2016; Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez, 2017). Our results show that dispersion of the marginal return of capital across firms (MPK) also increase for the credit bust after 2011, and we relate the MPK dispersion increase with the loan growth at narrowly defined sector level. In a recent paper, Kalemli-Özcan, Laeven, and Moreno (2018) documents that debt overhang is instrumental in accounting for the large drop in investment for non-financial corporations in the European debt crisis. Using the same dataset, we show debt overhang dampens firm’s capital growth in response to productivity growth.

The remainder of this paper is structured as follows. In the following section, we introduce the layout of the baseline model, the equilibrium characterization, as well as the discussions on policy implementations. In section 3, we provide industry and firm-level facts on debt overhang and capital reallocation, and we calibrate a quantitative industry dynamics model. Section 4 concludes.

2 The Model

In this section, we present the model layout, solve the competitive equilibrium and discuss inefficiency. The model consists of equal measure of identical households and ex-ante identical entrepreneurs. Both types of agents live for three dates: $t_0$, $t_1$ and $t_2$. There are two goods in the economy, capital (for example: land, building, and machinery, which we denoted as
and consumption (for example: food, cloths, which we denoted as $c$). Households and entrepreneurs have access technology to produce consumption goods using capital, but not the other way around. Both types of goods can be stored without depreciation across dates. Capital fully depreciates at date $t_2$. The model does not have labor, so capital is the only input for production.

2.1 Model Setting

**Endowments, Preferences and Technologies.** Households are representative. At $t_0$, each of them is endowed with $k_h$ unit of capital goods as well as $e_h$ unit of consumption goods. We assume $e_h$ is large enough so that they are unconstrained in equilibrium. Households do not receive endowment afterwards. The preference of is assumed to be linear without time discount,

$$U_h = c_{0h} + c_{1h} + c_{2h},$$

$c_{jh}, j \in \{0, 1, 2\}$ are the consumption levels in each date. At $t_0$, households operate a common decreasing technology that delivers consumption at $t_2$,

$$y_h = k_h^\gamma,$$

where $k_h$ is the capital inputs and $0 < \gamma < 1$ measures the degree of decreasing return to scale. As we discuss, the assumption that households’ revenue arrives at $t_2$ is not essential. Because households do not discount across periods and are assumed to be unconstrained, the production revenue can be realized in either of the three dates, and will yield the same equilibrium.

The setting of entrepreneur is as follows. At $t_0$, they are identical. Each of them is endowed with $k_e$ unit of capital (asset in place), and there is no consumption endowment. Similar with the households, they also do not receive any endowments at future dates. At $t_0$, they decide how much additional capital to invest for production. At $t_1$, their idiosyncratic productivity is realized: each of them operates a technology that delivers final consumption at $t_2$,

$$y = Az_j k,$$

where $A$ is a parameter that measures the aggregate productivity. $z_j \in \{z_H, z_L\}$, $z_H > z_L$, are the idiosyncratic productivity. The probability of becoming productive is $\pi$. The rest remains unproductive. Conditional on the realization of $z_j$, entrepreneurs trade capital with each other. Note that households do not trade capital at this date because capital is specific to the entrepreneur sector after investment is made at $t_0$. 


Assumption 1 (Idiosyncratic Uncertainty). *Entrepreneur needs to invest at $t_0$ before the realization of their idiosyncratic productivity at $t_1$.*

This timing assumption says input for production must be made under idiosyncratic uncertainty (Asker, Collard-Wexler, and De Loecker, 2014; Benhabib, Wang, and Wen, 2015; David, Hopenhayn, and Venkateswaran, 2016). In other words, capital input is pre-determined.

Entrepreneur only consumes at $t_2$. Their expected utility at $t_0$ is given by

$$U_e = \max_i E(c_2),$$

where $i$ is the initial investment and the expectation is taken over their idiosyncratic productivity realization. As aggregate technology $A$ is taken as given for now, the economy does not have aggregate uncertainties.

**Capital and Credit Markets.** Capital is traded at $t_0$ and $t_1$. At $t_0$, households and entrepreneurs trade capital at competitive price $P_0$. At $t_1$, entrepreneurs trade capital with each other at competitive price $P_1$. Denote $i$ as the quantity of investment purchased from the households at $t_0$, then $k \equiv k_e + i$ is the total capital at hand when uncertainty resolves at $t_1$.

As entrepreneurs do not have consumption endowment, they need external financing at both date $t_0$ and $t_1$. We assume that entrepreneurs cannot issue equity, so that the only way to finance capital purchase is to use debt contract. In both periods, credit market (or financial intermediaries) are competitive, and receives deposits from lenders and lends out to borrowers. No time discount imply unit interest rate across periods.

The friction of the credit market is entrepreneurs have limited commitment. They may default on loans when production revenue is realized at $t_2$. When this happens, entrepreneurs steal $1 - \theta$ fraction of total revenue, and leaves the remaining $\theta$ fraction, as well as their debt behind. We make the following assumption on debt seniority rule,

**Assumption 2 (Debt Overhang).** *Existing debt is senior to new debt.*

The above assumption says new creditors will have low priority relative to existing ones in case of liquidation (Hart and Moore, 1995; Philippon, 2009). In our model, this assumption is equivalent to say: if entrepreneur default at $t_2$, date $t_0$ creditor will gets paid first, and date $t_1$ creditor gets the remaining. As we discuss, it give rise to debt overhang.

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5Entrepreneurs can have a small amount of non-verifiable revenue at $t_1$. In this case, the ex-post unproductive ones will choose to save this revenue because it only consumes at $t_2$. The productive entrepreneurs are indifferent between early payment of $t_0$ debt and using the money to purchase capital. As we show, two decision have identical effect on entrepreneurs’ borrowing constraint, and thus profit at $t_2$. In this case, however, intermediate output need to be small enough to prevent perfect capital reallocation.
2.2 Competitive Equilibrium

Households. We first solve the household’s problem. As there is no uncertainty, we have

$$U_h = \max_{c_{jh}, \phi_h} c_{0h} + c_{1h} + c_{2h},$$  \hspace{1cm} (1a)$$

where $c_{jh}$ is date $t_j$ consumption and $\phi_h$ is the fraction of household capital to supply on the capital market at $t_0$. The flow of funds constraint is

$$c_{0h} + c_{1h} + c_{2h} = e_h + P_0 \phi_h k_h + [(1 - \phi_h) k_h]^\gamma,$$

total consumption equals to the total endowment $e_h$, profit from supplying capital $P_0 \phi_h k_h$, and the production revenue $[(1 - \phi_h) k_h]^\gamma$. The fact that households does not discount across dates keeps interest equals to 1 and they are indifferent in which periods to consume. Households maximization gives the following capital supply

$$k_h - \phi_h k_h = \left( \frac{P_0}{\gamma} \right)^{\frac{1}{\gamma-1}},$$  \hspace{1cm} (1b)$$
as $P_0$ increase, supply of capital $\phi_h k_h$ increases.

Entrepreneurs. We focus on ex-post first. At $t_1$, investment $i$ has been decided and productivity is realized. Conditional on the productivity draw, entrepreneur’s problem is to choose the amount of capital to trade. The optimization program for each type of entrepreneurs is given as follows,

$$c_H = \max_{1 + \phi_H \geq 0} A z_H (1 + \phi_H) k - (P_0 i + P_1 \phi_H k), \hspace{1cm} (2a)$$

$$c_L = \max_{1 - \phi_L \geq 0} A z_L (1 - \phi_L) k - (P_0 i - P_1 \phi_L k), \hspace{1cm} (3a)$$

recall $k \equiv k_x + i$ is the capital before trading; $\phi_H$, $\phi_L$ denote fraction of capital traded for the productive/unproductive respectively; $(1 + \phi_H) k$ and $(1 - \phi_L) k$ is the capital used for production; $P_0 i$ is the total amount debt incurred at $t_0$ (recall $P_0$ is the price, and $i$ is the quantity); $P_1 \phi j k$, $j \in \{H, L\}$ is the borrowing incurred at $t_1$ to facilitate capital trading. We put a negative sign in front of $\phi_L$ because, as we show, both $\phi_L$ and $\phi_H$ will be positive in equilibrium: capital reallocates from the unproductive entrepreneur to the productive ones. The following figure summarize the time-line.
Because borrower can steal up to $1 - \theta$ fraction of final output. The no-default constraint guarantee that paying debt delivers higher value to borrower than default, that is

$$Az_H (1 + \phi_H) k - (P_0 i + P_1 \phi_H k) \geq (1 - \theta) Az_H (1 + \phi_H) k$$

this give rise to the following debt overhang constraint

$$\frac{P_1 \phi_H k}{\text{new debt}} \leq \frac{\theta Az_H (1 + \phi_H) k - P_0 i}{\text{liquidation value} - \text{outstanding debt}}, \quad (2)$$

from here we can see how a large amount of ex-ante debt reduces ex-post borrowing capacity. A hypothetically exogenous increase in $P_0$ leads to a lower $\phi_H$. The unproductive have a similar problem,

$$c_L = \max_{1 - \phi_L \geq 0} Az_L (1 - \phi_L) k - (P_0 i - P_1 \phi_L k)$$

and as we show, they will not be constrained in equilibrium. The following figure shows entrepreneur’s balance sheet ex-ante and ex-post.
Because of linear assumption on entrepreneurial technology, we get the decision rule on capital trading,

\[
\phi_H = \begin{cases} 
-1 & \text{if } P_1 > A_z H \\
\frac{\theta A_z H k - P_0 i}{(P_1 - \theta A_z H) k} & \text{if } P_1 = A_z H \\
\frac{\theta A_z H k - P_0 i}{(P_1 - \theta A_z L) k} & \text{if } P_1 < A_z H
\end{cases}
\] (3)

and

\[
\phi_L = \begin{cases} 
1 & \text{if } P_1 > A_z L \\
\frac{-\theta A_z L k-P_0 i}{(P_1 - \theta A_z L) k} & \text{if } P_1 = A_z L \\
\frac{-\theta A_z L k-P_0 i}{(P_1 - \theta A_z L) k} & \text{if } P_1 < A_z L
\end{cases}
\] (4)

We now derive capital supply and demand function. As entrepreneurs trade capital between themselves, the supply has to comes from unproductive ones. When \( P_1 = A_z L \), they are indifferent between buying/selling capital; when \( P_1 > A_z L \), they supply all their capital. Hence, the total capital supplied on \( t_1 \) market, normalized by \( \pi K \equiv \pi(K_e + I) \), the total amount of
capital at the hand of ex-post productive entrepreneurs\(^6\), is given by

\[
\begin{align*}
\text{if } P_1 &= A \zeta_L \\
\theta \frac{A \zeta_H K - P_0 I}{(P_1 - \theta A \zeta_H) K} &\text{ if } P_1 < A \zeta_H \\
\in \left[0, \frac{1 - \pi}{\pi}\right] &\text{ if } P_1 = A \zeta_H
\end{align*}
\]

\(2b\)

similarly, capital demand comes from productive ones

\[
\begin{align*}
\theta \frac{A \zeta_H K - P_0 I}{(P_1 - \theta A \zeta_H) K} &\text{ if } P_1 < A \zeta_H \\
\in \left[0, \frac{1 - \pi}{\pi}\right] &\text{ if } P_1 = A \zeta_H
\end{align*}
\]

\(3b\)

note borrowing constraint \((2)\) binds when \(P_1 < A \zeta_H\).

Figure 3 graphs the supply and demand schedule, together with equilibrium level aggregate productivity. As long as \(P_1 > A \zeta_L\), low productive entrepreneurs supply all of their capital, and capital reallocation is complete.

We proceed to describe entrepreneurs’ ex-ante problem. At \(t_0\), individual entrepreneur choose investment \(i\) to solve

\[
\max_i \pi c_H + (1 - \pi) c_L
\]

conditional on \(I, P_0, P_1\).

**Competitive Equilibrium.** We define the competitive equilibrium as follows

**Definition 1.** The competitive equilibrium is a quadruple \((i, I, P_0, P_1)\) that satisfies the following conditions:

1. conditional on \(I, P_0, P_1\), households and entrepreneurs solve \((1a)-(4a)\).

2. \(I, P_0, P_1\) are determined by equilibrium conditions \((1b)-(3b)\).

3. \(i = I\).

before solving the equilibrium, we make the following parametric assumption

**Assumption 3.** It is assumed that

\[
\frac{k_h - \left(\frac{A \zeta_H}{\gamma}\right)^{\frac{1}{\gamma - 1}}} {k_h + k_e - \left(\frac{A \zeta_H}{\gamma}\right)^{\frac{1}{\gamma - 1}}} \leq \theta \leq (1 - \pi) \frac{z_L}{z_H},
\]

\(5\)

\(^6\)We use upper case letter for aggregate values and use lower case letter to denote individual value hereafter.
Figure 3: Ex-post Equilibrium

\[ P_1 \]

\[ A_{z_H} \]

\[ A_{z_L} \]

\[ (1 - \pi)A_{z_L} + \pi A_{z_H} \]

\[ \Phi_H \]

\[ \frac{1 - \pi}{\pi} \]
and that

$$K_h > \left( \frac{A_z L}{\gamma} \right)^{\frac{1}{\gamma - 1}}$$  \hspace{1cm} (6)

We explain the intuition for the assumption as follows: as $\theta$ determines ex-post debt capacity, equation (5) says the ex-post productive entrepreneurs can pledge sufficient amount to outside investors so that there are strictly positive capital reallocation in equilibrium (the left hand side), but at the same time not too much so that all capital reallocates from unproductive to productive (the right hand side). In other words, this assumption enables us to solely focus on the situation when capital reallocation is positive but incomplete, as in Figure 3. Note that the right hand side of equation (5) also implies $z_L > \theta z_H$, financial constraint indeed binds. Equation (6) says entrepreneurs are productive enough so that investment level $i > 0$ in equilibrium. We relegate the discussions on the cases when the above assumptions are violated to the Appendix A.2.

We now solve the equilibrium. Substituting equation (2a) into equation (4a), and notice $P_1 = A z_L$ implies $c_L = A z_L k - P_0 i$. This gives

$$\pi c_H + (1 - \pi) c_L = \pi [A z_H (1 + \phi_H) k - (P_0 i + P_1 \phi_H k)] + (1 - \pi) (A z_L k - P_0 i)$$

recall ex-post capital decision of the productive in equation (3), the above equation can be re-written as

$$\left\{ \begin{align*}
\pi A z_H + (1 - \pi) A z_L + \pi \frac{z_H - z_L}{z_L - \theta z_H} \left( \theta A z_H - P_0 \frac{i}{i + k_e} \right) \\
\text{financial autarky} \\
\text{reallocation}
\end{align*} \right\} (i + k_e) - P_0 i$$  \hspace{1cm} (7)

We discuss intuition. Suppose ex-post credit market is shut down, then each entrepreneur simply use their own capital to produce. In this case, the return to installed capital is the average productivity: $\pi A z_H + (1 - \pi) A z_L$. The fact that the capital brought into period $t_1$ can be reallocated implies this return also depends how much productive entrepreneurs can leverage ex-post, namely, their debt capacity. As entrepreneurs making investment decision ex-ante, they are aware that their investment will induce debt overhang of their own: $i/(i + k_e)$. But, what they did not internalize is the fact that their investment, as a whole, raises equilibrium capital price $P_0$, which squeezes the debt capacity of every productive entrepreneurs ex-post, weakens reallocation.

We analyze the source of externality. At $t_1$, the marginal value of entrepreneurial wealth depend on idiosyncratic realization of productivity. In low state, one more unit of investment increases utility by $A z_L$; In high state, it is $A z_H + A (z_H - z_L) \left( \frac{\theta A z_H - P_0}{(z_L - \theta z_H) A} \right)$, where the first term
is production revenue, and the second term is leveraged return, in which \( P_0 \) is determined in general equilibrium. Intuitively, agents want to carry wealth into good state, where their return from investment is negatively related with aggregate investment. In Lorenzoni (2008), the corresponding formula is \((1 - \theta)A_{q - \theta A}\), where \( q_s \) is the ex-post fire sale price. Under the classification in Dávila and Korinek (2018), the current model belongs to the “distributive externality” category. The following proposition shows the competitive equilibrium.

**Proposition 1** (Competitive Equilibrium). Under assumptions (1) – (3), the competitive equilibrium \((I^*, P^*_0, P^*_1)\) is

\[
I^* = K_h - \left(\frac{P^*_0}{\gamma}\right)^{\frac{1}{\gamma - 1}},
\]

\[
P^*_0 = A\frac{\pi z_H + (1 - \pi) z_L + \pi z_H - \pi z_L \theta z_H}{1 + \pi z_H - \theta z_H},
\]

\[
P^*_1 = Az_L,
\]

capital reallocates from the unproductive to the productive but the process is incomplete, i.e. \(0 \leq \Phi_H \leq 1 - \pi\).

**Proof.** see Appendix A.1. □

### 2.3 Constrained Efficiency and Optimal Policy Mix

The planner is constrained in the following sense: it can only choose the amount of investment \( I \) (via some policy instruments as we discuss), leaving all other decisions to private agents; Equilibrium prices are determined by market forces as well. Therefore, the only difference between competitive equilibrium and constrained efficient one is the fact that planner know her decision \( I \) will affect equilibrium prices and allocations (Stiglitz, 1982; Geanakoplos and Polemarchakis, 1985; Dávila and Korinek, 2018). Depending on whether the constrained planner puts weight on household sector or not, the constrained efficient allocation is defined as follows.

**Definition 2.** If the planner put zero weight on households, he constrained efficient allocation triplet \((I^{**}, P^{**}_0, P^{**}_1)\) satisfies

1. planner choose \( I^{**} \) to maximize utility in (4a), subject to (1a)–(3a), and (1b)–(3b).

2. \( P^{**}_0, P^{**}_1 \) determined by equilibrium conditions (1b)–(3b).

and if the planner put equal weight on households, \((I^{**}, P^{**}_0, P^{**}_1)\) satisfies
1. planner choose $I^{**}$ to maximize utility that adds (1a) and (4a), subject to (1a)–(3a), and (1b)–(3b).

2. $P_0^{**}, P_1^{**}$ determined by equilibrium conditions (1b)–(3b).

We solve the first case. Conditional on planner’s choice in $I^{**}$, and market price $(P_0^{**}, P_1^{**})$, entrepreneurs’ themselves solve equation (2a), (3a). Plugging their decisions in $c_H$ and $c_L$ into equation (4a) yields the problem for planner,

$$\max_{I} \left\{ \pi A z_H + (1 - \pi) A z_L + \pi A z_H - P_1 \left( \theta A z_H - P_0 \frac{I}{I + K_e} \right) \right\} (I + K_e) - P_0 I, \quad (11)$$

subject to equations (1b), (2b), (3b). Under the parametric assumption, it is clear that $\theta$ is small enough so that $P_1 = A z_L$ also holds in planner’s case. Substituting this into the above maximization problem, taking first order condition, and using the capital supply equation gives

$$P_0^{**} = A \frac{\pi z_H + (1 - \pi) z_L + \pi A z_H - z_L - \theta A z_H}{1 + \frac{\pi z_H - z_L - \theta A z_H}{1 + \frac{1}{\varepsilon}}} \quad (12)$$

where $1/\varepsilon$ is the inverse of price elasticity of capital supply (evaluated at $P_0^{**}$),

$$\frac{1}{\varepsilon} = I_0 \frac{dP}{dI_0} = (1 - \gamma) \frac{I^{**}}{K_h - I^{**}}, \quad (13)$$

as $1/\varepsilon > 0$, over-borrowing is clear. Following the same analysis, if planner weights households equally,

$$P_0^{**} = A \frac{\pi z_H + (1 - \pi) z_L + \pi A z_H - z_L - \theta A z_H}{1 + \frac{\pi z_H - z_L - \theta A z_H}{1 + \frac{1}{\varepsilon}}} \quad (14)$$

where $1/\varepsilon$ is the same with equation (13). Hence, comparing with the competitive equilibrium, social planner will invest/borrow strictly less in both cases. But comparing planner’s first and the second case, her investment will be higher in the second one. The intuition is direct: as a higher price increase the welfare for households, if the planner weights on them, it will not let ex-ante price/investment going too low. The following proposition summarizes the constrained efficient allocation.

---

7Due to the static nature of our model, if the planner choose ex-ante borrowing/investment via a macro-prudential policy instrument, it cannot revise ex-post. Hence, there is no time-inconsistency issue. In dynamic settings, this might not be the case, see Bianchi and Mendoza (2018) for discussions.

8In the appendix, we discuss the case when $\theta$ is larger than the range set by assumption 3. As $\theta$ increases to the level in which the competitive equilibrium close to complete capital reallocation, the planner face a subtle decision: if the misallocation is costly enough, say $z_H/z_L$ very large, the planner would choose to reduce investment to a level in which there is no misallocation ex-post, i.e. $P_1 > A z_L$; on the other hand, if misallocation not costly, the planner will choose investment so that $P_1 = A z_L$. The technical reason for this is change in $I$ leads to non-differentiability in planner’s value function due to non-differentiability in $P_1$. The discussions is given in Appendix A.2. For now, the parametric assumption makes sure $P_1 = A z_L$ holds in planner’s allocation.
Proposition 2 (Constrained Efficient Allocation). In constrained efficient allocation, $P_{0}^{**}$ is given by equation (12) if the planner assign zeros weight on households, and is given by equation (14) if the planner put equal weights. In both cases, $I^{**}$ satisfies

$$K_{h} - I^{**} = \left( \frac{P_{0}^{**}}{\gamma} \right)^{\frac{1}{\gamma - 1}},$$

and $P_{1}^{**}$ satisfies

$$P_{1}^{*} = A z_{L},$$

the competitive equilibrium allocation is not constrained efficient, entrepreneurs over-borrow ex-ante, and the aggregate productivity is too low.

In what follows, we consider the case when social planner choose equal weight (Dávila and Korinek, 2018). Discussions on general Pareto weighting relegated to Appendix A.3.

We analyze the welfare gain from ex-ante de-leveraging. Consider a thought experiment at $t_0$: the economy is originally at the competitive equilibrium in which every entrepreneur choose $I^{*}$ and the investment is priced at $P_{0}^{*}$. Each entrepreneur is obliged to reduce investment by a very small amount of $\Delta I$. This indicates that entrepreneur’s ex-ante leverage will be reduced by the amount of $\frac{K_{h}}{(I + K_{e})^{2}} \Delta I$. At the same time, this de-leveraging process results in drop in asset price by $\Delta P_{0}$ (all $\Delta$s are in positive values to avoid confusion, we put negative signs in front if the variable is negative). The overall ex-ante welfare gain for entrepreneurs

$$\Delta U_{e} = A \left( \pi z_{H} + (1 - \pi) z_{L} + \pi \frac{z_{H} - z_{L}}{z_{L} - \theta z_{H}} \right) \Delta I$$

where we have used equation (9) to cancel terms in the first equality. For households,

$$\Delta U_{h} = -(I^{*} \Delta P_{0} + P_{0}^{*} \Delta I) + \gamma (K_{h} - I^{*})^{\gamma - 1} \Delta I$$

welfare lose from drop in capital price. Summing up $\Delta U_{e}$ and $\Delta U_{h}$ yields the ex-ante social welfare gain

$$\Delta U = \pi \frac{z_{H} - z_{L}}{z_{L} - \theta z_{H}} I^{*} \Delta P_{0}$$

clearly, the positive social welfare gain in ex-ante de-investment/de-leveraging comes from im-
proved aggregate efficiency. The tension between individual and group is that atomic individual will not choose to invest less because its decision does not affect price. Thus, this welfare improving de-investment/leveraging will not happen in the competitive equilibrium. We proceed to discuss policy implementation.

**Implementation.** There are potentially two types of policies to correct this inefficiency. On the one hand, the government could use quantity based policy such as limiting on loan-to-value (LTV) ratio; on the other hand, it could also use the price based policy such as tax borrowing. In practice, the quantity based policy are used more often than the price based rules because the latter is typically more costly to operate. The price based rules can, however, provide a gauge of the extent of the inefficiency: a larger tax rate indicates heavier regulatory burden (Jeanne and Korinek, 2013). The implementation of the LTV ratio is relatively straightforward, the government can simply set the limit that corresponds to that of the constrained planner’s case. For price based rules, we consider a proportionally tax on the borrower (entrepreneur) at $t_0$, and do a lump-sum transfer to each borrower (Jeanne and Korinek, 2013). Denote $\tau$ as the tax rate and $T$ as the lump-sum transfer. Given this policy, for each one dollar borrowed, the cost of borrowing raise to $1 + \tau$. Suppose the entrepreneur choose to invest in the amount of $i$, her $t_1$ payment now becomes $(1 + \tau) (P_0 i - T)$. This gives the $t_0$ problem for entrepreneurs,

$$
\max_i \left\{ \left\{ \pi A z_H + (1 - \pi) A z_L + \pi \frac{z_H - z_L}{z_H - \theta z_H} \left( \theta A z_H - \frac{(1 + \tau)(P_0 i - T)}{i + k_e} \right) \right\} (i + k_e) \right\} - (1 + \tau) (P_0 i - T)
$$

under this policy, the equilibrium capital price is given by,

$$
P_0 = \frac{\pi z_H + (1 - \pi) z_L + \pi \frac{z_H - z_L}{z_L - \theta z_H} \theta z_H}{(1 + \frac{z_H - z_L}{z_L - \theta z_H}) (1 + \tau)} A
$$

thus we have the following proposition.

**Proposition 3 (Implementation).** The ex-ante loan-to-value (LTV) limit to implement the constrained efficient allocation is given by

$$
\varphi = \frac{I^{**}}{I^{**} + K_e},
$$

where $I^{**}$ is the planner’s investment level. The optimal tax rate on the ex-ante entrepreneur is given by

$$
\tau = (1 - \gamma) \frac{\pi \frac{z_H - z_L}{z_L - \theta z_H}}{1 + \pi \frac{z_H - z_L}{z_L - \theta z_H}} \frac{I^{**}}{K_h - I^{**}}
$$
Proof. note φ is the planner’s loan-to-value ratio; and τ is solved by equating (16) with (14).

**Proposition 4** (Comparative Statics). It can be shown that

\[
\frac{d\tau}{dA} > 0, \quad \frac{d\tau}{d\pi} > 0
\]

namely, the implied wedge is larger in productivity booms.

Proof. it suffices to notice *I*∗∗ is an increasing function of both A and π.

The tax rate on borrowing τ provide the measure of the gap between market’s and planner’s allocation. First, τ is positively related with \(\pi z \frac{\alpha - \gamma}{\alpha - \gamma} z_H \): as there are more productive entrepreneurs operating, reducing the ex-ante asset price simply can make more entrepreneurs better off by reducing their debt burden. That is, a larger τ is needed. Second, τ is also positively related with the inverse of asset supply elasticity: \((1 - \gamma) \frac{I^{**}}{K_h - I^{**}}\): for the kind of capitals whose supply schedule is inelastic (for example, land (Saiz, 2010)), the gap between competitive equilibrium and constrained efficient one is larger, because price will increase faster with investment demand.

**Aggregate Uncertainty.** In this section, we compare ex-ante and ex-post policy measures. To this end, we introduce aggregate shocks into the modelling economy. The source of aggregate uncertainty is on the realization of the aggregate productivity, A. Assuming that A takes random values, which is realized at date \(t_1\), simultaneously with the idiosyncratic productivity shocks after investment is made. Applying pricing equation (7) gives

\[
P^{*0} = \frac{\pi z_H + (1 - \pi) z_L + \pi \frac{\alpha - \gamma}{\alpha - \gamma} z_H \theta z_H}{1 + \pi \frac{\alpha - \gamma}{\alpha - \gamma} z_H} \mathbb{E} (A),
\]

and the planner’s problem can be solved as follows

\[
P^{**0} = \frac{\pi z_H + (1 - \pi) z_L + \pi \frac{\alpha - \gamma}{\alpha - \gamma} z_H \theta z_H}{1 + \pi \frac{\alpha - \gamma}{\alpha - \gamma} z_H \left(1 + (1 - \gamma) \frac{I^{**}}{K_h - I^{**}}\right)} \mathbb{E} (A),
\]

Note that with aggregate uncertainty, \(P^{*0}\) and \(P^{**0}\) will be functions of macro expectation: \(\mathbb{E} (A)\). The ex-post reallocation efficiency, \(\Phi_H\) now depends on the realization of aggregate productivity shocks, with \(\Phi_H (A) = \frac{\theta z_H A - P^{*0} \frac{I}{K_h - I^{**}}}{(z_L - \theta z_H) A}\), we thus have

\[
\Phi' (A) = \frac{1}{(z_L - \theta z_H) A^2 I + K_e} P^{**0} I,
\]

and we have the following Proposition followed by a Figure that illustrate this Proposition.
**Proposition 5** (Excessive Volatility). *Reallocation efficiency is pro-cyclical, and exhibit excessive volatility in the competitive equilibrium.*

**Proof.** note that pro-cyclicality comes from $\Phi'_H(A) > 0$; and excessive volatility comes from $I$ and $P_0$ are larger in competitive equilibrium than that of the social planner’s. ■

Figure 4: Graphical Illustration of Reallocation Cyclicality

\[ \Phi(A) = \frac{\theta A_H - P_0 I}{A_L - \theta A_H} \]

**Ex-post Interventions.** Following Jeanne and Korinek (2013), we consider two commonly used ex-post policies: debt bailout and monetary stimulus. In our setting, debt bailout means the government bailout entrepreneur’s ex-ante debt using the tax payer’s money. This can be seen as a debt relief program that is proportional to the outstanding stock of debt. The monetary stimulus, on the other hand, is to subside new borrowing: a subsidy on the collateral asset. As is discussed in Farhi and Tirole (2012), such an intervention could be interpreted as a monetary relaxation that lowers the real interest rate. We will be specific about the setting in the following discussion.

We first focus on ex-post debt bailout. In particular, for every 1 dollar worth of ex-ante debt, the government bailout $s$ ($s < 1$). In this policy, the cost is borne by the savers in the economy, i.e. the households. The ex-post redistribution however, incur a dead-weight loss in the amount of $\gamma(x)$, where $x$ is the amount distributed from the households to the entrepreneurs. Given this, we can write the entrepreneur’s problem at date $t_0$ as a function of the bailout ratio $s$,

\[
\max \begin{cases} 
(\pi \mathbb{E}(A) z_H + (1 - \pi) \mathbb{E}(A) z_L)(I + K_e) \\
+\pi \frac{z_H - z_L}{z_L - \theta z_H} [\theta z_H (I + K_e) \mathbb{E}(A) - (1 - s) P_0 I] - (1 - s) P_0 I \end{cases}
\]
this gives the capital price as a function of \( s \)

\[
P_0^*(s) = \frac{\pi z_H + (1 - \pi) z_L + \pi \frac{z_H - z_L}{z_L - \theta z_H} \theta z_H}{(1 - s) \left(1 + \pi \frac{z_H - z_L}{z_L - \theta z_H}\right)} E(A),
\]

the above equation says the following: while a positive bailout rate can alleviate the debt burden of the productive entrepreneurs ex-post, it can push up the ex-ante return to capital because entrepreneurs are forward-looking. The increase in \( P_0^*(s) \) and \( I^*(s) \) will reduce ex-post reallocation efficiency

\[
\Phi_H(s) = \pi \frac{z_H - z_L}{z_L - \theta z_H} \left( \theta A z_H - (1 - s) P_0^*(s) \frac{I^*(s)}{I^*(s) + K_e} \right),
\]
as it gets smaller than that of the competitive equilibrium.

Second, we consider monetary stimulus. In this case, the government subsidize new borrowing by a rate of \( \rho \). The ex-post constraint is given by

\[
P_1 \phi_H k \leq (1 + \rho) [\theta A z_H (1 + \phi_H) k - P_0 i],
\]

this means the productive entrepreneurs can leverage by more

\[
\phi_H \leq \frac{(1 + \rho) \left(\theta A z_H - P_0 \frac{i}{1 + k_e}\right)}{A z_L - (1 + \rho) \theta A z_H}
\]

and using the entrepreneur’s pricing equation yields

\[
P_0^*(\rho) = \frac{\pi z_H + (1 - \pi) z_L + \pi (1 + \rho) (z_H - z_L) \frac{\theta z_H}{z_L - (1 + \rho) \theta z_H}}{1 + \pi (1 + \rho) \frac{z_H - z_L}{z_L - (1 + \rho) \theta z_H}} E(A)
\]
as \( P_0^*(\rho) \) is a decreasing function of \( \rho \), the monetary stimulus works differently comparing with the debt bailout. While both of the policies expands the debt capacity of the ex-post productive, they create different ex-ante incentive. Debt bailout acts as subsidy on ex-ante borrowing while monetary acts as tax: as an entrepreneurs borrows more ex-ante, it will get more subsidy under debt bailout, and less subsidy under monetary stimulus.

Greenspan doctrine says macro-prudential policy is superfluous and only mopping up after the crisis is necessary. A few papers shows that ex-post interventions could distort ex-ante incentive (Farhi and Tirole, 2012). We obtain similar results here, ex-post bailout worsen the over-borrowing problem. The new result here is the two types of interventions provide opposite ex-ante incentives through debt overhang.
**Optimal Policy Mix.** Real world policy makers access to a wide range of ex-ante and ex-post policies for crisis management. This section characterize the optimal mix of ex-ante and ex-post policy interventions of a benevolent social planner that maximizes welfare in the economy. We first solve for the optimal policy mix of a discretionary planner. We then proceed to show this solution is consistent with the the one under commitment. In other words, optimally set policy mix is time consistent.

We specify the policy instrument as follows. Ex-ante, there are two macroprudential policies available: the policy maker can directly set a debt cap (maximum loan-to-value limit) to reduce borrowing; alternatively, the planner can implement a propotional tax on ex-ante debt: for every dollar borrowed, entrepreneur needs to pay $1 + \tau$, the tax revenue $T$ are rebated in a lump-sum fashion. Ex-post, government can subsidize new lending at rate $\rho_m$, or mop up $\rho_b$ fraction of existing debt (Jeanne and Korinek, 2013). As ex-post policies involves resource redistribution, we denote $g(x)$ as the deadweight loss of $x$ unit of consumption goods redistributed: $g(0) = 0$, and $g'(x) > 0$. Our discussion of ex-ante policy focus propotional tax on borrowing first. As we show, debt cap can be equivalently implemented through planner’s optimality condition. In what follows, we define and solve the competitive equilibrium under discretionary policy.

**Definition 3** (Discretionary policy mix). The competitive equilibrium under discretionary policy is a Markov perfect equilibria, in which the social planner’s date-1 policies $(\rho_m, \rho_b)$ are contingent on the realization of $A$, as well as macroprudential tax $\tau$. The competitive equilibrium consists of

1. real allocations $(i(\tau), \phi_H(\rho_m, \rho_b, \tau, A), \phi_L(\rho_m, \rho_b, \tau, A))$
2. asset prices $P_0(\tau), P_1(\rho_m, \rho_b, \tau, A)$
3. date-0 policy $\tau$ and date-1 policy $(\rho_m, \rho_b)$

at both date-0 and date-1,

1. households and entrepreneurs maximizes utility subject to budget and financial constraint
2. social planner maximizes welfare

The equilibrium is solved backwards. We assume redistribution of public funds is costly enough so that policy instruments in equilibrium cannot fully solve capital misallocation problem. Given this, $P_1 = Az_L$. To productive entrepreneurs, the budget constraint changes to

$$P_1 \phi_H k \leq (1 + \rho_m) [\theta A z_H (1 + \phi_H) k - (1 - \rho_b)(1 + \tau) (P_0 i - T)],$$

(19)
where $1 + \tau$ is macroprudential tax and $T$ is the lump-sum rebate. At this stage, both $\tau$ and $T$, together with $P_0$, $i$ are taken as given. Clearly, both $\rho_m$ and $\rho_b$ increase borrowing capacity by raising $\phi_H$. Because $P_1 = A z_L$, the borrowing constraint binds for productive entrepreneur. Given there is ex-post misallocation, $P_1 = A z_L$, the above budget constraint binds. In aggregation

$$
\Phi_H (\rho_m, \rho_b, I, A) = \frac{(1 + \rho_m) \left[ \theta A z_H - (1 - \rho_b) \frac{(1 + \tau) (P_0 I - T)}{I + K_e} \right]}{A z_L - (1 + \rho_m) \theta A z_H},
$$

(20)

this gives the ex-post problem of the discretionary planner, who choose the ratio of subsidy $\rho_m$, $\rho_b$ to maximize welfare

$$
W (I, A) = \max_{\rho_m, \rho_b} \left\{ \pi c_H (\rho_m, \rho_b, I, A) + (1 - \pi) c_L (\rho_m, \rho_b, I, A) - g (\rho_m, \rho_b, I, A) \right\}
$$

(21)

$$
= \max_{\rho_m, \rho_b} \left\{ \begin{array}{c}
A \left[ \pi z_H + (1 - \pi) z_L + \pi (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A) \right] (K_e + I) \\
- g (\rho_m, \rho_b, I, A) - (1 - \rho_b) (1 + \tau) (P_0 I - T) - \rho_b (1 + \tau) (P_0 I - T)
\end{array} \right\}
$$

first order condition on $\rho_m$ and $\rho_b$ gives

$$
\pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial \rho_j} (K_e + I) = \frac{\partial g (\rho_m, \rho_b, I, A)}{\partial \rho_j} + \lambda_j, \; j \in \{m, b\},
$$

(01)

where $\lambda_j$ denotes the Lagrange multiplier on constraint: $\rho_j \geq 0$. The complementary slackness conditions are given by

$$
\lambda_j \rho_j = 0, \; \lambda_j \geq 0,
$$

(02)

note that the choice of $\rho_j$ is conditional on the realization of $A$.

Ex-ante, entrepreneurs solves

$$
\max_i \left\{ \mathbb{E} \left[ \pi A z_H + (1 - \pi) A z_L + \pi A (z_H - z_L) \phi_H \right] (i + k_e) - \mathbb{E} \left[ (1 - \rho_b) (1 + \tau) (P_0 i - T) \right] \right\},
$$

where $\phi_H$ is given in equation (19). As $\rho_b$ is determined ex-post, the expectation is taken over $(1 - \rho_b) (1 + \tau) (P_0 i - T)$, the payment of ex-ante debt. F.o.c. on $i$ gives $P_0$ as a function of $\tau$. The maximization problem implies $P_0$ as a function of $\tau$. The following proposition states this result.

**Proposition 6 (Asset Price).** Given policy mix $(\tau, \rho_m, \rho_b)$, asset price $P_0$ in competitive equi-
librium is given by

\[
P_0 = \frac{\mathbb{E} \left\{ A \left[ \pi z_H + (1 - \pi) z_L + \frac{\pi(1+\rho_m)(z_H-z_L)\theta z_H}{z_L-(1+\rho_m)\theta z_H} \right] \right\}}{(1 + \tau) \left( \mathbb{E} [1 - \rho_b] + \mathbb{E} \left[ \frac{\pi(1+\rho_m)(1-\rho_b)(z_H-z_L)}{z_L-(1+\rho_m)\theta z_H} \right] \right)},
\]

(22)

crucially, \( P_0 \) only depends on macroprudential tax \( \tau \). By equation (1b), so is investment \( I \).

**Proof.** The pricing equation is from the first order condition of the above maximization program of entrepreneurs.

The reason for asset price only depend on macroprudential tax is the following: by definition, discretionary planner’s ex-post interventions \( \rho_m, \rho_b \) are optimal response of its ex-ante intervention \( \tau \), through which private incentive to borrow in response to ex-post interventions are corrected. In other words, planner’s ex-ante decision on \( \tau \) will internalize private agents’ optimal response towards ex-post intervention. The result here proves to be the key reason for discretionary policy to be time consistent.

Given the optimal ex-ante decision above, we are now ready to derive the planner’s program. It consists of the following elements

1. entrepreneur revenue \( \mathbb{E} \left[ \pi A z_H + (1 - \pi) A z_L + \pi A (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A) \right] (I + K_e) \), net expected payment of ex-ante debt \( \mathbb{E} \left[ (1 - \rho_b) (1 + \tau) (P_0 I - T) \right] \)

2. households revenue \( P_0 I + (K_h - I)^\gamma \), net debt bailout money \( \mathbb{E} [\rho_b (1 + \tau) (P_0 I - T)] \).
   Note monetary stimulus only incur redistributional cost, because this debt is paid by entrepreneurs eventually.

3. ex-post intervention in the form of debt bailout and monetary stimulus incur cost from redistribution \( \mathbb{E} [g (\rho_m, \rho_b, I, A)] \)

Adding up the three items above gives planner’s ex-ante problem

\[
\max \tau \left\{ \mathbb{E} \left[ A \left[ \pi z_H + (1 - \pi) z_L + \pi (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A) \right] (I + K_e) \right] + (K_h - I)^\gamma \right\}
\]

(23)

in deriving the above program, we have plugged planner’s budget balance condition: \( T = \frac{\tau}{1+\tau} P_0 I \). As we are solving for the discretionary planner’s problem, the only choice variable here is \( \tau \), and takes into account the optimal response of \( \rho_m, \rho_b \) as a function of \( \tau \). Taking first
order condition with respect to $\tau$ and use equation (o1) to cancel out terms\(^9\)

$$
\mathbb{E}\left[ [\pi A z_H + (1 - \pi) A z_L + \pi A (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A)] + \pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial \rho_j} (I + K_e) - \frac{\partial g(\rho_m, \rho_b, I, A)}{\partial \rho_j} \right] = \gamma (K_h - I)^{\gamma - 1} \quad (o3)
$$

so that $\rho_m$, $\rho_b$, and $I$ (equivalently $\tau$ by equation (22)) are jointly determined by equation (o1), and (o3).

We now solve for policy mix under commitment, namely, the case of a Ramsey planner. The only difference is planner choose the full set policy $(\tau, \rho_m, \rho_b)$ to maximize social welfare, and commit to its policy decision ex-post. In particular, the planner solve the following problem

$$
\max_{\tau, \rho_m, \rho_b} \left\{ \mathbb{E}\left[ A [\pi z_H + (1 - \pi) z_L + \pi (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A)] (I + K_e) \right] + (K_h - I)^{\gamma} \right\}
$$

$\rho_m$, $\rho_b$ should maximize the term inside expectation bracket $\mathbb{E} [\cdot]$. That is, for each realization of $A$, the following first order condition satisfies

$$
\pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial \rho_j} (K_e + I) = \frac{\partial g(\rho_m, \rho_b, I, A)}{\partial \rho_j} + \lambda_j, \; j \in \{m, b\}
$$

together with the complementary slackness conditions. First order condition on $\tau$

$$
\mathbb{E}\left[ A [\pi z_H + (1 - \pi) z_L + \pi (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A)] + \pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial I} (I + K_e) - \frac{\partial g(\rho_m, \rho_b, I, A)}{\partial I} \right] = \gamma (K_h - I)^{\gamma - 1}
$$

note that above two equation is identical with equation (o1) to (o3). Hence the discretionary solution is the same with the one under commitment.

\(^9\)Note that

$$
\gamma (K_h - I)^{\gamma - 1} \frac{\partial I}{\partial \tau} = \mathbb{E}\left\{ [\pi A z_H + (1 - \pi) A z_L + \pi A (z_H - z_L) \Phi_H] \frac{\partial I}{\partial \tau} \right\} + \pi A (z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_m} \frac{\partial I}{\partial \tau} + \pi A (z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_b} \frac{\partial I}{\partial \tau} + \pi A (z_H - z_L) \frac{\partial \Phi_H}{\partial I} \frac{\partial I}{\partial \tau} (K_e + I)
$$

plugging equation (o1) into the expectation bracket gives

$$
\gamma (K_h - I)^{\gamma - 1} \frac{\partial I}{\partial \tau} = \mathbb{E}\left\{ [\pi A z_H + (1 - \pi) A z_L + \pi (z_H - z_L) \Phi_H] \frac{\partial I}{\partial \tau} + \pi A (z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_m} \frac{\partial I}{\partial \tau} (K_e + I) - \frac{\partial g(\rho_m, \rho_b, I, A)}{\partial I} + \lambda_m \frac{\partial \rho_m}{\partial \tau} + \lambda_b \frac{\partial \rho_b}{\partial \tau} \right\}
$$

note $\lambda_j \frac{\partial \rho_j}{\partial \tau} = 0, \; j \in \{m, b\}$. The constraint either (1) not binding, in which case the multiplier $\lambda_j = 0$, or (2) the constraint binds, in which case $\frac{\partial \rho_j}{\partial \tau} = 0$. In the former case, the Lagrange multiplier is zero, in the latter case, $\rho_j$ does not respond to $\tau$. The equation in text can be obtained by divides $\frac{\partial I}{\partial \tau}$ term from both sides.
Proposition 7 (Optimal policy mix). The optimal policy mix is a triplet \((\tau, \rho_m, \rho_s)\) that consists of macroprudential tax \(\tau\), ex-post monetary stimulus \(\rho_m\), and debt bailout \(\rho_s\). The three variables jointly solves equations \((o1)\) to \((o3)\). Moreover, equilibrium allocation under discretion agrees with the one under commitment.

Proof. proof is given in the discussion above.

We discuss the time-consistent result here. The key reason for this result, according to the discussion in Proposition (6), is that macroprudential tax can be optimally implemented. Because of this, planner’s optimal decision on macroprudential tax effectively regulate private agents’ ex-ante incentive to borrow towards ex-post interventions.

We provide an example as follows. Suppose the dead-weight loss takes a quadratic form: \(g(x) = \psi x^2 / 2\), where \(\psi\) is a large constant so that planner itself, under discretionary or commitment, cannot have capital completely reallocated. In other words, competitive equilibrium with optimal policy mix features capital misallocation ex-post. With this, we show that the optimal policy mix \((\rho_m, \rho_b, \tau)\) satisfies (solution is given in Appendix A.6)

\[
\rho_b = 0 \tag{24}
\]

\[
\psi \frac{\pi \rho_m}{1 + \rho_m} A z_L \Phi_H (\rho_m, \rho_b, I, A) K = \frac{z_H - z_L}{z_L - \theta z_H} \tag{25}
\]

\[
E \left\{ \pi A z_H + (1 - \pi) A z_L + \pi \frac{(1 + \rho_m) (z_H - z_L)}{z_L - (1 + \rho_m) \theta z_H} \left[ \theta A z_L - (1 - \rho_b) \left( \frac{\partial P_0}{\partial I} I + I P_0 \right) \right] \right\} \tag{26}
\]

\[
= \gamma (K_h - I)^{\gamma - 1}
\]

equation (24) says it is optimal not to use debt bailing out: for each one dollar redistributed, only \(\pi\) goes to the productive entrepreneur. In standard externality models with fire sale, social planner ex-post is indifferent between choosing debt bailout and monetary stimulus (Lorenzoni, 2008; Jeanne and Korinek, 2013), because the two types of policy have identical effect on the representative entrepreneur’s flow of funds constraint at date-1. Equation (25) equalizes the cost of public funds to the benefit of improving asset reallocation. Finally, equation (26) is the first order condition with respect to macroprudential tax \(\tau\). According to equation (25), the total amount of public fund redistribution \(\frac{\pi \rho_m}{1 + \rho_m} A z_L \Phi_H (\rho_m, \rho_b, I, A) K\) is a constant, hence equation (26) does not involve terms on the cost of redistribution \(g(x)\). For a numerical illustration, Figure 5 shows the welfare comparison under first-best, macroprudential policy only, and optimal policy mix.
Figure 5: Welfare comparison

Note: the parameter that we use for this figure is $\pi = 0.20$, $z_H = 1.1$, $z_L = 0.9$, $\psi = 1.0$, $\gamma = 0.10$, $K_h = 2.00$, $K_e = 1.00$. The distribution of $A$ takes two values: 1.1 and 0.9, with equal probability.

Sub-optimal Macroprudential Policy. In practice, macroprudential tax oftentimes faces restriction, so that its optimal level cannot be achieved. The reason for this includes insufficient legal mandate, or the presence of a shadow banking sector. To facilitate discussion, we denote by $\tau_c$ the constrained level, and we focus on the commitment type first. In this case, as $\tau_c$ is given, the planner choose $I$, and state dependent ex-post intervention $(\rho_m, \rho_b)$ to maximize the following Lagrange program

$$
\mathcal{L} = \mathbb{E} \left[ A \left( \pi z_H + (1 - \pi) z_L + \pi (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A) \right) (I + K_e) ight] + (K_h - I)^\gamma 
$$

$$
+ \chi \left\{ \gamma (K_h - I)^{-1} - \frac{\mathbb{E} \left[ A \left( \pi z_H + (1 - \pi) z_L + \frac{\pi(1+\rho_m)(z_H-z_L)\theta z_H}{z_L-(1+\rho_m)\theta z_H} \right) \right]}{(1 + \tau_c) \left( \mathbb{E} [1 - \rho_b] + \mathbb{E} \left[ \frac{\pi(1-\rho_b)(1+\rho_m)(z_H-z_L)}{z_L-(1+\rho_m)\theta z_H} \right] \right)} \right\}
$$

where $\chi$ is the multiplier on the equality constraint which is essentially the asset pricing equation of the private agent in Proposition (6). This is the constraint that the planner faces when choosing the level of $I$, as the private agents are making decisions on borrowing and investment.
First order condition on $I$ gives
\[
E \left[ A \left[ \pi z_H + (1 - \pi) z_L + \pi (z_H - z_L) \Phi_H (\rho_m, \rho_b, I, A) \right] + \pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial I} (I + K_e) - \frac{\partial g (\rho_m, \rho_b, I, A)}{\partial I} \right]
- \gamma (K_h - I)^{\gamma - 1} + \chi (1 - \gamma) \gamma (K_h - I)^{\gamma - 2}
= 0
\]

note that by equation (o3), $\chi > 0$ as long as the investment level is higher than the case when macroprudential policy is unrestricted. First order condition on $\rho_m, \rho_b$ gives
\[
\pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial \rho_j} (K_e + I) = \frac{\partial g (\rho_m, \rho_b, I, A)}{\partial \rho_j} + \lambda_j - \chi \frac{\partial \Omega}{\partial \rho_j}; \quad (27)
\]
where
\[
\Omega \equiv A \left[ \pi z_H + (1 - \pi) z_L + \frac{\pi (1 + \rho_m) (z_H - z_L) \theta z_H}{z_L - (1 + \rho_m) \theta z_H} \right]
\]
\[
\left( 1 + \tau_c \right) (1 - \rho_b) \left[ 1 + \pi \frac{(1 + \rho_m) (z_H - z_L)}{z_L - (1 + \rho_m) \theta z_H} \right]
\]
and $I$ is given by the equality constraint:
\[
\gamma (K_h - I)^{\gamma - 1} = \frac{\mathbb{E} \left\{ A \left[ \pi z_H + (1 - \pi) z_L + \frac{\pi (1 + \rho_m) (z_H - z_L) \theta z_H}{z_L - (1 + \rho_m) \theta z_H} \right] \right\}}{(1 + \tau_c) \left( \mathbb{E} [1 - \rho_b] + \mathbb{E} \left[ \frac{\pi (1 - \rho_b) (1 + \rho_m) (z_H - z_L)}{z_L - (1 + \rho_m) \theta z_H} \right] \right)}, \quad (28)
\]
Under discretionary policy, we have
\[
\pi A (z_H - z_L) \frac{\partial \Phi_H (\rho_m, \rho_b, I, A)}{\partial \rho_j} (K_e + I) = \frac{\partial g (\rho_m, \rho_b, I, A)}{\partial \rho_j} + \lambda_j \quad (29)
\]
whereas the investment level $I$ is similarly determined in equation (28). The presence of term $\chi \frac{\partial \Omega}{\partial \rho_j}$ in equation (27) thus creates a wedge between the optimal allocation under commitment and discretion when macroprudential tax cannot operate at its optimal level. Recall our previous discussion for ex-post interventions, the following condition holds,
\[
\frac{\partial \Omega}{\partial \rho_m} < 0, \text{ and } \frac{\partial \Omega}{\partial \rho_b} > 0,
\]
namely, the presence of debt overhang imply opposite incentives for subsidizing new debt and bailing out old debt. By comparing equation (27) and (29), optimal ex-post intervention should involve less debt bailing out of existing debt and more subsidy on new debt. The following Proposition states this result.

**Proposition 8** (Sub-optimal Macroprudential Policy). If macroprudential tax $\tau$ is restricted
below its optimal level, then
\[ \rho^c_m \geq \rho^d_m, \text{ and } \rho^e_j \leq \rho^d_j \]
where \( \rho^e_j, j \in \{m, b\} \) denote the ex-post intervention under commitment, and \( \rho^d_j \) are the ones under discretion.

Limitation on macroprudential tax means restrictions on discretionary planner’s ability to reduce private agents’ ex-ante borrowing incentives in response to ex-post intervention. This create room for commitment: engaging more generous subsidy on new debt and less subsidy on existing debt effectively reduces ex-ante incentive to borrow.

3 Micro Data Facts and Model Quantification

Micro Data Facts. In this section, we provide empirical facts on corporate debt and the potential capital misallocation. The data that we use for empirical exercise is the ORBIS-Amadeus, a pan European dataset that have a representative coverage for private firms. The dataset we use is the same with that in Gopinath et al. (2017). This dataset have several advantages for the current empirical exercise. First, over 99% of the firms in this dataset are private firms that is financed exclusively using debt, which enables me to avoid capital structure related concerns; Second, the most detailed industry classification in this data is the 4 digit NACE (Nomenclature générale des Activités économiques dans les Communautés Européennes). More important, as mentioned by Reis (2013, 2015), the experience of Europe and its periphery in the 2000s and the great financial crisis that follows is unique in terms of looking for clues on corporate borrowing and productivity. Despite its rich coverage, our data per se, do not allow us to identify causal relationship between debt overhang and capital reallocation. We nevertheless highlight three robust patterns on the role of debt overhang.

Our facts here highlight the role of long-term debt, as long-term debt is the one that typically subject to overhang. We provides the related facts as follows.

Fact 1. Firms within manufacturing industry holds more long-term debt than short-term.

First, we document that long-term debt accounts for the vast majority of firm debt borrowed from banks. Entrants and young firms holds strikingly more. Figure 9 shows this pattern: as firm enters the market, the composition of long-term debt over total debt borrowed from the bank is around 80% for entrants and declines over age.

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Footnote: Statistical classification of economic activities in the European Communities. It has around 230 sub-sectos within the manufacturing industry. Thus have comparable level of detail the to 6 digit NAICS (North American Industry Classification System), which have around 270 sub manufacturing sectors. Detailed data cleaning processes identical with Gopinath et al. (2017), and the summary statistics reported in the Appendix C.
Fact 2. Given a narrowly defined manufacturing industry, countries that rely more on long-term financing is associated with larger increase in the dispersion of marginal return to capital.

In the environment of Hsieh and Klenow (2009), the dispersion of the marginal productivity of capital (MPK) measures the allocation inefficiencies that arises from firm-level idiosyncratic distortions for capital input. Gopinath et al. (2017) adopt this accounting framework to document that, at a narrowly defined industry level, the MPK dispersion for the southern European sovereigns has been steadily increasing since 2000. Their exercise mainly focus on the period of credit boom, i.e. before the 2011 European debt crisis. In Figure 10, we plot the post crisis trajectory for MPK dispersion at 4 digit NACE level. To make the comparison consistent, we replicate the exercise in Gopinath et al. (2017) and only extend to a longer series. In particular, we calculate the variance across firms in a given industry s and year t. Second, for each year, we calculate the dispersion across all sectors as the weighted average of dispersion. Each industry is assigned by an time-invariant weight that corresponds to the average manufacturing value added. For the same NACE 4 sectors, we sort countries by their long-term debt/total debt ratio for the year 2009, the year in which the crisis starts. We observe two patterns. First, the increasing trend of MPK dispersion before the crisis also persists after the crisis. Within the framework of Hsieh and Klenow (2009), this is puzzling: if the dispersion of return to capital is mainly drive by credit force\cite{footnote11}, then theories that imply capital mis-allocation in credit booms should predict improved reallocation efficiency in credit bust. Second, in this figure, sectors that rely more on long-term financing before the crisis increase their MPK dispersion by more after the crisis. The divergence persists until 2015. In order to address the concern that the trend here might be instead driven by firm churning, the lower panel reports the result when the dispersion is computed on the set of firms that exists in the sample period continuously (see Data Appendix for details). Under permanent sample, we obtain a smaller magnitude than that of the permanent sample.

Fact 3. As firms accumulates debt with longer maturities, capital responds less strongly with positive productivity innovations.

Our specification follows the standard Fazzari, Hubbard, Petersen, Blinder, and Poterba (1988) approach and we add firm-level leverage variable. Our benchmark specification takes the following form

$$\Delta k_{jt} = \alpha_0 \varepsilon_{jt-1}^{TFP} + \alpha_1 \varepsilon_{jt-1}^{TFP} \times \text{leverage}_{jt-1} + \alpha_2 \text{leverage}_{jt-1} + \alpha_3 X_{jt-1} + \eta_i,s,t + \varepsilon_{jt},$$

where $\Delta k_{jt}$ is the percentage change of year $t$ capital stock measured as $\Delta k_{jt} = 2 \times \frac{(k_{jt} - k_{jt-1})}{(k_{jt} + k_{jt-1})}$.

Capital stock are the sum of tangible fixed capital and intangible fixed capital. $\varepsilon_{jt}^{TFP}$ is the

\cite{footnote11}Admittedly, other mechanism might play important role: uncertainty, fixed cost, firm specific policy distortions, etc.
firm-level TFP innovation that is obtained from regressing firm’s current period \( \log(\text{TFP}) \) on its lagged level, controlling for firm fixed effect. \( \Delta \text{leverage}_{j,t-1} \) is the firm-level leverage measured by the logarithm of loan(-1)/sales(0). \( \mathbf{X}_{j,t-1} \) includes the standard controls: firm size \((k_{j,t-1})\), lagged capital change \((\Delta k_{j,t-1})\), sales growth, etc. Crucially, we includes the industry×country×year fixed effect to capture any industry level shocks (for example, industry specific demand/uncertainty/credit shock).

Table 1 shows the regression results. First, \( \alpha_0 > 0 \) indicates that firm capital grows faster when there is a positive productivity shock. However, this positive relationship is largely dampened as the firm becomes more leveraged. We then separately look at each firm-level leverage: trade credit, short-term loan, and long-term loan. In the Amadeus data for our investigated years, trade credit have an average “maturity” of around 30 days. The very short maturity suggests that is not likely to be subject to the problem of debt overhang. As expected, we obtain a positive relationship interaction term between trade credit and TFP innovation. In contrary, we obtain a strong negative relationship on long-term debt.

**Industry Dynamics and Model Quantification.** This section intends to provide an estimation of the magnitude of the debt overhang externality. To this end, we study a firm dynamics model a la Hopenhayn (1992). In the model, entrepreneurs enters and exits the industry at a constant rate \( \lambda \), in which they produce homogeneous consumption goods. Before entry, entrepreneurs are identical and their productivity is realized after entry. Production inputs are capital and labor. Labor decision is unconstrained while capital decision is constrained by the total amount of short-term debt the entrepreneur can issue.

Upon entry, entrepreneur’s start-up capital are finance by the issue one single piece of long-term debt and they cannot issue afterwards. The source of debt overhang in the model comes from the fact that long-term debt issued at entry is senior than the short-term debt incurred afterwards. The assumption that entrepreneurs can only issue long-term debt at entry simplifies the equilibrium characterization by reducing the dimension of the problem. The payment structure of the long-term debt is given as follows: following the setting in Leland (1994), Hackbarth, Miao, and Morellec (2006), Miao and Wang (2010), the principal of this long-term debt follow geometric decay with rate \( \delta \), namely one dollar of debt at date \( t \) becomes \((1 - \delta)\) at date \( t + 1 \). At each date, the firm need to pay a constant coupon rate \( c \) if not exiting. If the firm exits, it need to pay the principal\(^{12}\). The payment structure is illustrated in Figure

\(^{12}\)First, competitive credit market suggests the return on long term bond should equals to the short term interest rate \( r \), that is \( 1 = \lambda + (1 - \lambda) \left( c + \frac{1}{1 + r} (1 - \chi) \right) \), a higher decay rate must be matched with a higher coupon rate; Second, we show that the model is able to quantitatively capture the ratio of long-term debt to short-term debt for entrants, which is the main driving force that lead to our quantitative prediction. The model mechanism should is robust to alternative specifications of long-term debt payment, conditional on the specification can generate patterns on long-term debt holdings for entrants. But in this case, the numerical
At each date $t$, the supply of new capital consistent with the static model,

$$K_h - I = \left( \frac{P_t}{\gamma} \right)^{\frac{1}{\gamma - 1}},$$

where entrant access to the new capital market to purchase start-up capital using long-term debt. We assume only entrants can have access to the new capital market while incumbents cannot. Our technological assumption here is in the spirit of Putty-Clay, in which capital are of different vintages. Once capital is installed, investment cannot be further made and only reallocation is possible.

Entrepreneurs productivity are realized once the start-up capital are purchased and installed. The realization of productivity $z_i \in \{z_1, ..., z_N\}$ is discrete and is drawn from a time invariant distribution $G(z)$. The productivity evolves stochastically afterwards following a $N$ state Markov Chain $\Gamma(z_i, z_j)$ that satisfies the usual conditions. Following Moll (2014), Midrigan and Xu (2014), we make the assumption that entrepreneur’s productivity is known one period ahead. This assumption make the capital decision static and thus reduces the problem dimension by one. The time-line for an entrepreneur that enters at date $t$ is given in Figure (7).

We first describe the incumbents’ problem. At date $t$, an entrepreneur of age $j$ that does not exit have the following budget constraint (for notation simplicity, we drop the age subscript $j$),

$$c_t + k_{t+1} - (1 - \delta) k_t = \max_{n_t \geq 0} \left\{ e^{z_t} p_{t}^\alpha n_t^\gamma - w n_t \right\} + \frac{b_{t+1}}{1 + r} - b_t - c_t,$$

where the entrepreneurs’ consumption $c_t$ and investment $k_{t+1} - (1 - \delta) k_t$ are financed by operating profit, short-term debt rolling over minus the coupon payment of long-term debt. $w$ is implementation can be highly burdensome.
the wage rate that is exogenously given. While labor decision \( n_t \) is not constrained, investment and consumption is constrained by the short-term debt capacity,

\[
b_{t+1} \leq \theta k_{t+1} - d_{t+1},
\]

because short-term debt is used to finance working capital, from the above equation, debt overhang induced by long-term borrowing, \( d_{t+1} \), reduce the short-term debt capacity. As only the productive entrepreneurs will be constrained, debt overhang depress reallocation efficiency.

An entrepreneur that exits the economy simply consume the process from revenue net debt payment and consumes the remaining cash flow

\[
c_t = \max_{n_t \geq 0} \{e^{\gamma t} k_t^{\alpha} n_t^{\nu} - w n_t\} + (1 - \delta) k_t - b_t - d_t,
\]

the following figure summarizes the timing of the entrepreneurs’ problem

Figure 7: Timeline

Our timing assumption that entrepreneurs know their productivity next period reduces the problem dimension by one, and enables us to write the problem into a more streamlined representation. We denote \( a_t \equiv (1 + r) k_t - b_t \) as net worth and write both the continuer’s and the exiter’s problem in the following recursive form. First, let \( \pi_t \) be the operating profit

\[
c_t + \frac{a_{t+1}}{1 + r} = \pi_t + a_t - c d_t,
\]

subject to

\[
k_{t+1} \leq \frac{1}{1 + r - \theta} (a_{t+1} - d_{t+1})
\]

(31)
and the Bellman equation is given by

\[ V_j(a_t, d_t, z_t) = \max_{\{a_{t+1}, d_{t+1}, z_{t+1}\}} \{ \log c_t + \beta [(1 - \lambda) V_{j+1}(a_{t+1}, d_{t+1}, z_{t+1}) + \lambda W_{j+1}(a_{t+1}, d_{t+1}, z_{t+1})] \} \]

where \( W_{j+1}(a_{t+1}, d_{t+1}, z_{t+1}) \) is the value of the exiter and is equals to \( \log c_{t+1} \), where

\[ c_{t+1} = \pi_{t+1} + a_{t+1} - d_{t+1}. \]

Finally, the problem of the date \( t \) entrant is to choose the investment \( i \), in order to maximize their expected utility

\[ \bar{V} = \max_i \left\{ \int [(1 - \lambda) V_0(a_t, d_t, z_t) + \lambda W_0(a_t, d_t, z_t)] \, dz_t \right\}, \] (32)

where the first term is the value function conditional on not exiting and the second term is when the firm exits after entry. The choice of \( i \) increase net worth but increase long-term borrowing

\[ a_t = (1 + r)(k_e + i_t), \]

and

\[ d_t = P_t i_t, \]

where first equation is from the definition of \( a_t \) with \( k_e \) being the capital endowed by entrant firms; the second simply says entrepreneurs needs to use long-term debt to finance its initial investment expenditure.

Before proceeding to the model quantification, we show qualitative difference between the competitive equilibrium and that of the social planner. We focus on the steady state, and we drop time subscript to simplify notation.

**Proposition 9.** Let \( I^* \) and \( I^{**} \) denote the amount of investment for the competitive equilibrium and the social planner at the steady state

\[ I^* \geq I^{**} \]

**Proof.** the proof is given in Appendix A.7. ■

We provide three remarks on the model layout. First, the assumption of a Putt-Clay technology introduces capital irreversibility and vintage specific capital productivity. It is widely used in business cycle analysis (Gilchrist and Williams, 2000). The economic interpretation here is that start-up investment can be seen as new machines/production lines. Each period a set of new “projects” becomes available, and the investment productivity is realized only when the
investment is fully installed. In addition, old machines can only be reallocated but cannot add new investment until it is been replaced. Second, interest rate and wage rate are fixed in our model, and we show there are mainly two reasons for this. The main implications due to this assumption is that there is no interaction between different vintages. On the one hand, the planner’s program specified above is equivalent with the one that maximizes the steady state social welfare. On the other hand, making interest rate and wage endogenous introduces another type of externality, as investment decisions will spill-over across different capital vintages through wage and interest, and complicates our analysis of the externality that is present in this model (Blanchard and Fisher, 1989). Third, the liquidation value when an firm exits the economy is exogenous. In other words, there is no “fire-sale” in the standard sense. Although this setting is in the spirit of Moll (2014), Midrigan and Xu (2014), introducing “fire-sale” a la Kiyotaki and Moore (1997) would amplify our externality result because excessive borrowing ex-ante will make ex-post asset price “too low”.

We are not able to obtain analytical solution here, and we solve this model with numerical method. We relegate the numerical procedure to Appendix B and we provide an analytically tractable linear example of this model in Appendix A.8. We describe the parameterizations of the competitive equilibrium. The first set of parameter are directly taken from the literature. First, we set the discount rate \( \beta \) to be 0.95 and the inter-period risk free rate \( r \) to be 0.02 as in Gopinath et al. (2017). For the span-of-control parameter, we choose \( \alpha + \nu \) to be 0.85 as in Basu and Fernald (1997), and the capital elasticity to be 1/3. The rest of parameters are calibrated to match the data moments that are constructed from the ORBIS-Amadeus dataset. In particular, we choose \( \lambda \) to target a entry/exit rate of 0.04. Following Midrigan and Xu (2014), we choose the productivity persistence and dispersion in order to target the firm-level sales auto-correlation and the standard deviation of firm-level sales growth. We choose the financial constraint parameter \( \theta \) to match firm-level median (short-term) loan to sales ratio. Finally, \( k_e \) the capital endowment at startup is calibrated to match the entry firm’s size, measured as the relative tangible fixed capital to industrial average. The capital supply elasticity \( \gamma \) is choosen to match the entry firm’s leverage (short-term loan over sales) relative to industry mean. For the parameter that governs the average maturity of the long-term loan, we choose \( \delta \) in order to target a average maturity of about 5 years (EUROSTAT). All parameters above are shared for the social planner’s case when we solve for the constrained efficient allocation.

**Mechanism and Quantitative Implications.** The calibration results is shown in Table 3, and we highlight the moments related with long-term debt. First, our model setting here implies the firms repays its long-term debt at the first few years after entry. In the model, the long-term debt holding for average entrant at age 1 is about 65%, and this number drops to around 22% at age 5. In the data, the fraction of long-term debt holding decrease more
slowly. One reason is firms in reality can issue multiple long-term debt, this is particularly true for entering firms that have high growth potential. In the model, we put the restriction that entrepreneurs can only issue when they start business for numerical tractability.

Aggregate efficiency depend on how fast the productive firms grow. Now, suppose at date \( t \), \( \theta_{t+1} \) drops unexpectedly. Consider the budget constraint of an ex-post productive firm with age \( j \)

\[
c_t + k_{t+1} - (1 - \delta) k_t = \pi_t - \frac{cd_t}{\text{debt service}} - \frac{b_{t+1}}{1 + r_t} - b_t
\]

the peculiar externality raise price of capital and consequently long-term debt burden of all firms. It introduce two margins that affect growth of productive firms. First, increase debt service, act as tax on young (because of geometric decay) and productive (because they are binding). Drehmann, Juselius, and Korinek (2018) provides macro evidence. With geometrical decaying long-term debt, the coupon payment is given by

\[
 cd_t = c (1 - \upsilon)^j d_{t-j} \uparrow
\]

second, long-term debt constrains short-term debt capacity

\[
 b_{t+1} \leq \theta_{t+1} A_{t+1} z_{t+1} k_{t+1} - d_{t+1} \downarrow
\]

The holding of long-term debt induces the problem of debt overhang. In the decentralized economy, firms invests too much and borrows excessively upon entry. As a result, we show this lead to about 9% drop in employment growth in our calibrated model.

Finally, we conduct two counterfactuals when the calibrated economy receives aggregate shocks. The first one is when the economy is hit by a collateral shock, in which \( \theta \) drops by 20% and recovers follows an AR(1) process (Jermann and Quadrini, 2012). The result is as expected, the measured productivity upon impact drops by around 4.5% for the decentralized case. For the constrained efficient case, the drop is 3.7%. The gap persists after the tightening of collateral constraint \(^{13}\).

The second transition exercise is more related with case of the GIIPS nations (Greek, Italy, Ireland, Portugal, Spain). Following the introduction of EURO, the Sothern European are received a large amount of capital inflow. Reis (2013, 2015) provides evidence on Portugal, but the capital flow facts holds for all the GIIPS countries, with their average 10 year government bond rate drops from about 8% down to about 2% with the introduction of EURO (Bond Rate

\(^{13}\)The solution to the competitive equilibrium follows Guerrieri and Lorenzoni (2017); The solution to the constrained planner follows Bianchi (2010), in which the social planner choose the sequence of investment during the transition and takes the shock as given. The detailed description is given in Appendix. Here we follow Jeanne and Korinek (2013) and omit the discussion of time consistency issue as in Bianchi and Mendoza (2018).
The model exercise proceed as follows: we first simulate the economy at the steady state before the introduction of EURO, in which interest rate is given at $r = 8\%$. In the first shock, we let the interest rate drop to 2\%. The capital price $P_t$ along the transition path are based on the belief that the interest rate shock will be permanent. The second shock raise interest rate to 6\% permanently. In credit boom periods, competitive equilibrium features overaccumulation of long-term debt, and the economy exhibit excessively sullying. Debt overhang makes the productive young firms grows particularly slow when the interest rate suddenly increases. As a result, the measured productivity keeps to fall despite the interest rate rises. Here, the workings of the model is consistent with the empirical documentation in Drehmann, Juselius, and Korinek (2018): long-term debt decay slowly, and the debt service increases after several years of new debt accumulation. Although the interest rate increase in 2009 drive out the unproductive firms, the dispersion of return to capital across firms still increase because net worth drops due to debt service and debt overhang.

4 Conclusions

This paper proposes a model of firm over-borrowing with debt overhang. It offers a novel mechanism in which ex-ante policy measures are justified. The model highlight the role of asset price, debt overhang, reallocation and aggregate productivity.

Canonical models on firm overborrowing with financial constraints emphasis asset fire-sale induced by a binding collateral constraint (Shleifer and Vishny, 1992; Lorenzoni, 2008). In normal times, agents do not internalize the fact that their investment and borrowing decision makes asset price during downturn too low. In the model that we describe, competitive equilibrium do not have fire sale as date 1 asset price is a constant. We instead emphasis the general equilibrium force ex-ante, where firm investment result in indebtedness and also a boom of asset price.

By modelling debt overhang and firm heterogeneity explicitly, our model shed light on the fact of weak reallocation in aftermath of financial crisis. As we know, Japan in 1990s, the Great Recession, and the European Sovereign Debt Crisis are followed by not only large drop in asset price and investment but also a prolonged decline in productivity with persistently slow paces of reallocation (Caballero, Hoshi, and Kashyap, 2008; Kehrig, 2015; Foster, Grim, and Haltiwanger, 2016; Blattner, Farinha, and Rebelo, 2018b). In credit boom, asset price rises as debt piles up. As the boom gone bust, asset price collapses, but debt does not go away easily (Gomes, Jermann, and Schmid, 2016; Drehmann, Juselius, and Korinek, 2018). Indebtedness in credit boom years constrains the expansion of production units in bust as well, more so when they receive good investment opportunity (Blattner, Farinha, and Rebelo, 2018a). With productive units grow sluggishly and unproductive ones seeming not to shrink, reallocation
weakened, putting drags on the recovery of economy.

The model that we consider do not have financial intermediary. In reality, the surge of corporate borrowing, and the potential inefficiency embedded could also be driven by the collective behavior from the supply side. Moreover, macroprudential policies oftentimes work through financial intermediary, which, by the documentation of recent empirical literature, play an important role in transmitting these regulatory policies to the real economy (Blattner, Farinha, and Rebelo, 2018b). We leave the discussion on financial sector for future research.
References


—. 2013. “Macroprudential Regulation Versus Mopping Up After the Crash.”


This figure plots the pre-crisis change in corporate leverage and the post-crisis change in aggregate productivity; The data is downloaded from the OECD database. Debt is defined as a specific subset of liabilities: securities other than shares except financial derivatives, loans, and other accounts payable. Value added is the surplus (or deficit) from production before interest, rent or similar charges payable on financial or tangible non-produced assets. The non-financial corporation sector includes all private and public enterprises that produce goods and/or provide non-financial services to the markets. Productivity is the aggregate solow residual.
Figure 9: Firm Age and Long-term Financing

Raw data on long-term Debt/Total Bank Debt; Total bank debt is the sum of long-term and short-term. This result is computed by each country×industry and collapsed by the average across all country×industry.
First, we create a time-invariant NACE4 weight that is given by \( \frac{Y_s}{Y} \), the total (real) value added divided the value added of the whole manufacturing industry. Then, for each industry \( s \) and year \( t \), we calculate the dispersion across all sectors as the weighted average of dispersion. We do the same for the permanent sample, in which we only keep firms that exist in the panel continuously.
Table 1: firm-level Regression

<table>
<thead>
<tr>
<th></th>
<th>$\Delta k/k$</th>
<th>$\Delta k/k$</th>
<th>$\Delta k/k$</th>
<th>$\Delta k/k$</th>
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<tr>
<td>tfp innov.</td>
<td>0.0243***</td>
<td>0.0305***</td>
<td>0.0248***</td>
<td>0.0104***</td>
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<tr>
<td></td>
<td>(0.00225)</td>
<td>(0.00282)</td>
<td>(0.00442)</td>
<td>(0.00372)</td>
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<td>leverage</td>
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<td>-0.0103***</td>
<td>-0.0149***</td>
<td>-0.00662***</td>
</tr>
<tr>
<td></td>
<td>(0.000498)</td>
<td>(0.000571)</td>
<td>(0.000647)</td>
<td>(0.000731)</td>
</tr>
<tr>
<td>tfp innov. × leverage</td>
<td>-0.00552***</td>
<td>-0.00472***</td>
<td>-0.00751***</td>
<td>-0.00750***</td>
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<tr>
<td></td>
<td>(0.00116)</td>
<td>(0.00133)</td>
<td>(0.00177)</td>
<td>(0.00165)</td>
</tr>
<tr>
<td>tfp innov. × trade credit</td>
<td>0.00963***</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.00192)</td>
<td></td>
<td></td>
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<tr>
<td>tfp innov. × short debt</td>
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<td>-0.00150</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00140)</td>
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<td></td>
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<tr>
<td>tfp innov. × long debt</td>
<td></td>
<td></td>
<td>-0.00878***</td>
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</tr>
<tr>
<td></td>
<td></td>
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<td>(0.00171)</td>
<td></td>
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<tr>
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<td>1540292</td>
<td>1017983</td>
<td>1009216</td>
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<tr>
<td>$R^2$</td>
<td>0.336</td>
<td>0.347</td>
<td>0.370</td>
<td>0.379</td>
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<tr>
<td>firm level controls</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>firm fe</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>country × year × sector fe</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

TFP innov is measured productivity innovations; all R.H.S. vars are lagged; standard errors are clustered at firm-level and is reported in the parenthesis with ***,**,* denotes p value smaller than 10%, 5%, 1%, respectively.
<table>
<thead>
<tr>
<th>parameters</th>
<th>value</th>
<th>target and source</th>
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<td>exit rate</td>
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</tr>
<tr>
<td>prod. persis.</td>
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<td>sales auto-correlation</td>
</tr>
<tr>
<td>prod. innov. std.</td>
<td>0.091</td>
<td>std(sales growth)</td>
</tr>
<tr>
<td>borr. constr. (ent)</td>
<td>0.780</td>
<td>median loan/sales ratio</td>
</tr>
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<td>entry capital endow.</td>
<td>3.000</td>
<td>entry capt.</td>
</tr>
<tr>
<td>cap sup elas.</td>
<td>0.750</td>
<td>entry leverage</td>
</tr>
<tr>
<td>ltdb maturity</td>
<td>0.200</td>
<td>loan maturity</td>
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</table>

panel B: assigned parameters

<table>
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<td>Gopinath et. al. (2017)</td>
</tr>
<tr>
<td>risk-free rate</td>
<td>0.020</td>
<td>Gopinath et. al. (2017)</td>
</tr>
<tr>
<td>span-of-control</td>
<td>0.850</td>
<td>Basu and Fernald (1997)</td>
</tr>
<tr>
<td>capital share</td>
<td>0.333</td>
<td>standard 1/3</td>
</tr>
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</table>

Table 2: Parameter Assignment
Table 3: Moments

<table>
<thead>
<tr>
<th></th>
<th>model (decentral)</th>
<th>model (constrain)</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>panel A: targeted moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median firm age</td>
<td>16.000</td>
<td>16.000</td>
<td>14.000</td>
</tr>
<tr>
<td>naics4 entry rate</td>
<td>0.040</td>
<td>0.040</td>
<td>0.042</td>
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<tr>
<td>sales auto-correlation</td>
<td>0.857</td>
<td>0.863</td>
<td>0.890</td>
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<td>std(sales growth)</td>
<td>0.699</td>
<td>0.740</td>
<td>0.530</td>
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<td>median loan/sales ratio</td>
<td>1.887</td>
<td>1.640</td>
<td>1.930</td>
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<tr>
<td>entry capt</td>
<td>-1.412</td>
<td>-1.631</td>
<td>-1.850</td>
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<tr>
<td>entry leverage</td>
<td>0.842</td>
<td>0.653</td>
<td>0.514</td>
</tr>
<tr>
<td><strong>panel B: additional moments</strong></td>
<td></td>
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</tr>
<tr>
<td>fraction ltdb: age == 1</td>
<td>0.655</td>
<td>0.541</td>
<td>0.858</td>
</tr>
<tr>
<td>fraction ltdb: age == 2</td>
<td>0.524</td>
<td>0.432</td>
<td>0.805</td>
</tr>
<tr>
<td>fraction ltdb: age == 3</td>
<td>0.411</td>
<td>0.337</td>
<td>0.771</td>
</tr>
<tr>
<td>fraction ltdb: age == 4</td>
<td>0.322</td>
<td>0.214</td>
<td>0.729</td>
</tr>
<tr>
<td>fraction ltdb: age == 5</td>
<td>0.221</td>
<td>0.176</td>
<td>0.722</td>
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<tr>
<td>S.D. employment growth</td>
<td>0.723</td>
<td>0.753</td>
<td>0.303</td>
</tr>
<tr>
<td>S.D. capital growth</td>
<td>0.723</td>
<td>0.753</td>
<td>0.555</td>
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<tr>
<td>S.D. employment</td>
<td>1.349</td>
<td>1.362</td>
<td>1.324</td>
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<tr>
<td>S.D. capital</td>
<td>1.349</td>
<td>1.362</td>
<td>2.146</td>
</tr>
<tr>
<td>1-year autocorr. employment</td>
<td>0.857</td>
<td>0.871</td>
<td>0.970</td>
</tr>
<tr>
<td>5-year autocorr. employment</td>
<td>0.465</td>
<td>0.469</td>
<td>0.903</td>
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<tr>
<td>1-year autocorr. capital</td>
<td>0.857</td>
<td>0.875</td>
<td>0.974</td>
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<tr>
<td>5-year autocorr. capital</td>
<td>0.465</td>
<td>0.469</td>
<td>0.899</td>
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<tr>
<td>share producers, age 1-5</td>
<td>0.217</td>
<td>0.217</td>
<td>0.200</td>
</tr>
<tr>
<td>share output, age 1-5</td>
<td>0.227</td>
<td>0.175</td>
<td>0.105</td>
</tr>
<tr>
<td>share employment, age 1-5</td>
<td>0.228</td>
<td>0.172</td>
<td>0.096</td>
</tr>
<tr>
<td>share capital, age 1-5</td>
<td>0.228</td>
<td>0.179</td>
<td>0.081</td>
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<tr>
<td>rel. output growth 1-5 vs. 6+</td>
<td>0.233</td>
<td>0.251</td>
<td>0.136</td>
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<tr>
<td>rel. employ. growth 1-5 vs. 6+</td>
<td>0.239</td>
<td>0.261</td>
<td>0.102</td>
</tr>
<tr>
<td>rel. capital growth 1-5 vs. 6+</td>
<td>0.414</td>
<td>0.452</td>
<td>0.147</td>
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</table>

Panel A is targeted moments; Panel B is non-targeted moments; To construct data moments, we first compute for each country × industry and collapse by the median among them.
The upper panel plots the shock on firm level borrowing constraint $\theta_t$. It follows an log-AR(1) type, with 20% on impact and evolves with decay rate 0.80 (Jermann and Quadrini, 2012). The lower panel plots the response of of measured aggregate TFP, defined as $\log(Y_t) - \alpha \log(K_t) - \nu \log(N_t)$. Circledotted line is the decentralized case (competitive equilibrium), and the Triangle-dotted line is the constrained efficient case.
The upper panel is the average of long term bond rate for the GIIPS sovereigns (Bond Rate Link Here). Response of of measured aggregate TFP is defined as $\log(Y_t) - \alpha \log(K_t) - \nu \log(N_t)$. Circled-dotted line is the decentralized case (competitive equilibrium), and the Triangle-dotted line is the constrained efficient case.
A Proofs and Additional Discussions

A.1 Proof of Proposition

The pricing equation in this proposition is obtained by taking first order condition with respect to $i$ on equation (7). We need to check Assumption 3 indeed guarantees that there is positive but incomplete capital reallocation in equilibrium. First, notice the right hand side implies maximum amount of capital that an ex-post productive entrepreneur can finance is given by

$$\phi_H \equiv \frac{\theta A z_H - P_0 \frac{i}{i + k_e}}{P_k - \theta A z_H} \leq \frac{\theta A z_H}{A z_L - \theta A z_H},$$

where the above inequality use the fact that $P_0 \geq 0$ and $P_1 \geq A z_L$ should always holds. Using $\theta \leq \frac{1 - \pi}{\pi z_H}$, the above equation implies $\phi_H \leq \frac{1 - \pi}{\pi}$, hence in aggregation, $\pi \Phi_H \leq 1 - \pi$. Second, using the left hand side,

$$\phi_H \equiv \frac{\theta A z_H - P_0 \frac{i}{i + k_e}}{P_1 - \theta A z_H} \geq \frac{\theta A z_H - A z_H}{k_h + k_e - \left(\frac{A z_H}{\gamma}\right)^{1+1}} \geq 0$$

where in the second last inequality, we use the fact that $P_0$ should be smaller than the first best case $A z_H$ under all circumstances.

A.2 Assumption 3 is violated

We first discuss the case when the left hand side of equation (5) is violated, i.e. $\theta > (1 - \pi) \frac{z_L}{z_H}$. Under this case, there exists a threshold $\theta^*$ above which the first best allocation will be supported, namely, $P_0 = A z_H$ and $P_1 > A z_L$, and ex-post capital reallocation is complete. The cut-off condition for this is $\Phi_H = \frac{1 - \pi}{\pi}$, where $\Phi_H = \int \phi_H$. Given the first best case features $P_0 = A z_H$, the capital supply equation implies $I = K_h - \left(\frac{A z_H}{\gamma}\right)^{1+1}$. Using equation (3b), we have $\theta^*$ is given by $\frac{\theta^* z_L K - z_H}{(z_L - \theta^* z_H) K} = \frac{1 - \pi}{\pi}$. Solving for $\theta^*$ yields

$$\theta^* = (1 - \pi) \frac{z_L}{z_H} + \pi \frac{K_h - \left(\frac{A z_H}{\gamma}\right)^{1+1}}{K_h + K_e - \left(\frac{A z_H}{\gamma}\right)^{1+1}},$$

hence as long as $\theta > \theta^*$, the decentralized allocation reaches first best. The social planner’s solution under this case agrees with the decentralized market.
As

\[(1 - \pi) \frac{z_L}{z_H} < \theta < \theta^*,\]  

the decentralized market cannot support first best, so that \(I, P_0\) follows Proposition (1) in the text. But the social planner’s decision on \(I\) is slightly complicated. As \(I\) close to 0, \(P_1 > Az_L\) and \(\Phi_H = \frac{1-\pi}{\pi}\). This gives the value function for the social planner,

\[U(I) = Az_H I + (K_h - I)^\gamma\]  

but as \(I\) increases, \(P_1\) will go down, as in our discussion in ex-post equilibrium in figure (3). Because \(\theta < \theta^*\), \(P_1\) will reach \(Az_L\) before \(U(I)\) reaches its maximum. To see this, consider the situation when \(I_0^{**} = K_h - \left(\frac{Az_H}{\gamma}\right)^{1/\gamma}\) and \(P_0 = Az_H\). Then aggregate reallocation is given by

\[
\Phi_H = \frac{\theta Az_H K - Az_H I_0^{**}}{(P_1 - \theta Az_H) K} - \frac{\theta - \frac{I_0^{**} + K_e}{z_h - \theta}}{1 - \frac{z_L}{z_H} - \theta}
\]

\[
\leq \frac{K_h - \left(\frac{Az_H}{\gamma}\right)^{1/\gamma}}{K_h + K_e - \left(\frac{Az_H}{\gamma}\right)^{1/\gamma}}
\]

\[
\leq \frac{1 - \pi}{\pi}
\]

where the second inequality use \(P_1 > Az_L\); the third equation substitute \(I_0^{**}\) in; and the final inequality use the \(\theta < \theta^*\). This is a contradiction because \(P_0 = Az_H\), but reallocation is
incomplete. As the economy reaches $P_1 = Az_L$, social planner’s value function has a kink at this point, which we denote it as $I_1^{**}$. Around $I_1^{**}$, the slope of $U(I)$ drops discontinuously. This is because as $I$ increase, reallocation start to decline (note $\Phi_H'(I) = 0$ on left but $\Phi_H'(I) < 0$ on right). Depending on the magnitude of the drop, there are two possibilities: (1) if the drop is large enough, the optimum of $U(I)$ is simply $I_1^{**}$; in this case, misallocation is so costly that the planner would rather invest less; (2) if the drop is not large, the optimum of $U(I)$ will reach at some $I_2^{**} > I_1^{**}$; in this case, misallocation is not costly comparing with underinvestment at $I_1^{**}$; The analysis is illustrated with figure (15). Letting $\Phi_H = \frac{1-\pi}{\pi}$ gives $I_1^{**}$

$$\frac{I_1^{**}}{I_1^{**} + K_e} = \frac{1}{\pi} \left( \theta - (1 - \pi) \frac{z_L}{z_H} \right),$$

and $I_2^{**}$ takes the same format as in Proposition (2). Finally, the choice between $I_1^{**}$ or $I_2^{**}$ is given as follows

$$I^{**} = \arg\max \{U(I_1^{**}), U(I_2^{**})\},$$

and as $I_1^{**} < I_2^{**} < I^*$, over-borrowing holds in this case.

We now consider the case when the left hand side of equation (5) is violated. In this case, we denote another threshold $\theta_*$, below which the decentralized allocation features no ex-post reallocation. In this case, $P_0 = Az_L$. Using equation (3b), $\theta_*$ is given by

$$\theta_* = \frac{z_L}{z_H} \frac{k_h - \frac{Az_L}{A \gamma}}{k_e + k_h - \frac{Az_L}{A \gamma}},$$

as long as $\theta \geq \theta_*$, over-borrowing/investment holds. There are positive capital reallocation ex-post in both the competitive equilibrium and constrained efficient case.

A.3 Date $t_0$ Pareto weighting

For general Pareto weights $\rho$, $(0 \leq \rho \leq 1)$, we have the social welfare function

$$\max_I (1 - \rho) U_h + \rho U_e,$$

we have

$$P_0^{**} = A \frac{\rho \left( \pi A z_H + (1 - \pi) A z_L + \pi\theta A z_H z_L A_{z_H} z_L - \theta A z_H \right)}{\rho \pi A z_H - z_L - (1 - 2\rho) \left( 1 + \frac{1}{\gamma} \right) (1 - \rho)},$$

taking $\rho = 1$ and $\rho = 1/2$ yields equation (12) and (14). Note that as $\rho \rightarrow 0$, i.e. social planner does not value welfare of the entrepreneurs, the over-borrowing result may be overturned. It is intuitive: high price increase households’ welfare. This is related with the “anything goes”
result in Dávila and Korinek (2018).

### A.4 Date \( t_0 \) borrowing constraint

This appendix discusses the conditions under which the borrowers can default on date \( t_0 \) debt as well. Clearly, entrepreneur will not choose to default before its productivity is realized, because if they default ex-ante, \( k_e + i \) will be lost and there is no revenue yet. We can only consider the case when idiosyncratic productivity is realized. For high productive entrepreneurs. The no-default constraint implies

\[
Az_H (1 + \phi_H) k - \phi_H P_1 k - P_0 i \geq (1 - \theta) Az_H (1 + \phi_H) k,
\]

where the left hand side is the profit from no-default and the right hand side is the profit from default. The constraint is identical with borrowing constraint in equation (2). Therefore, high productive entrepreneurs at date \( t_2 \) will not be tempted to default on ex-ante debt. Applying the same analysis on the ex-post unproductive entrepreneurs, no-default constraint implies

\[
Az_L (1 - \phi_L) k + \phi_L P_1 k - P_0 i \geq (1 - \theta) \left[ \phi_L P_1 k + Az_L (1 - \phi_L) k \right],
\]

where the left hand side is the value of no-default and the right hand side is the value of default. Rewriting this equation gives:

\[
P_0 i \leq \theta \left[ Az_L (1 - \phi_L) k + \phi_L P_1 k \right],
\]

or

\[
P_0 \frac{i}{k_e + i} \leq \theta Az_L,
\]

this equation defines a cut-off value \( \theta^* \) below which the ex-ante debt will be constrained. Using the above equation and recall capital supply equation (1b), \( \theta^* \) can be determined as

\[
\theta^* = \frac{K_h - \left( \frac{P_0^*}{\gamma} \right) \frac{1}{1 - \gamma}}{K_h + K_e - \left( \frac{P_0^*}{\gamma} \right) \frac{1}{1 - \gamma}} \frac{P_0^*}{Az_L}
\]

where \( P_0^* \) is given in equation (9). If \( \theta \leq \theta^* \), debt issued at date \( t_0 \) would be a corner solution that is determined by the constraint. Equation (37), and the capital supply equation (1b) jointly gives \( P_0^*, I^* \). Given the planner respect this ex-ante constraint, there is nothing the social planner can do. After the entrepreneurs choice of \( i \) based on \( I, P_0 \) and \( P_1 \), the analysis of date \( t_1 \) decision will be unchanged.
A.5 Concavity in planner’s valuation

We notice that using the capital supply equation, the term

$$ \omega \equiv (1 - \gamma) I^{**} \left( \frac{P^*_0}{\gamma} \right)^{\frac{1}{1-\gamma}} $$

$$ = (1 - \gamma) \left( K_h \left( \frac{P^*_0}{\gamma} \right)^{\frac{1}{1-\gamma}} - \frac{P^*_0}{\gamma} \right) $$

total differentiating equation planner’s pricing equation yields

$$ \left( 1 + \pi \frac{z_H - z_L}{z_L - \theta z_H} \right) d\mathbb{E}[A] $$

$$ = \left( 1 + \pi \frac{z_H - z_L}{z_L - \theta z_H} \right) dP^{**} + (1 - \gamma) \left[ K_h \left( \frac{P^*_0}{\gamma} \right)^{\frac{1}{1-\gamma}} - 1 \right] dP^{**} $$

$$ = \left( 1 + \pi \frac{z_H - z_L}{z_L - \theta z_H} \right) + \frac{1}{\gamma} \left( \left( \frac{P^*_0}{\gamma} \right)^{\frac{1}{1-\gamma}} K_h - (1 - \gamma) \right) dP^{**} $$

thus

$$ \frac{dP^{**}}{d\mathbb{E}[A]} = \frac{1 + \pi \frac{z_H - z_L}{z_L - \theta z_H}}{1 + \pi \frac{z_H - z_L}{z_L - \theta z_H} + \frac{1}{\gamma} \left( \left( \frac{P^*_0}{\gamma} \right)^{\frac{1}{1-\gamma}} K_h - (1 - \gamma) \right)} $$

because $P^{**}_0$ is a increasing function of $\mathbb{E}[A]$, hence the above equation is a decreasing function of $\mathbb{E}[A]$. Thus it proves concavity.

A.6 Proof of the case when $g(x) = \psi x^2/2$

With $g(x) = \psi x^2/2$, ex-post maximization of the social planner is

$$ \max_{\rho_m, \rho_s} \left\{ \pi A (z_H - z_L) \Phi_H K - \frac{\psi}{2} \left( \pi \frac{\rho_m}{1 + \rho_m} Az_L \Phi_H K + \rho_s P_0 I \right)^2 \right\} $$

recall borrowing constraint (19), $\pi \frac{\rho_m}{1 + \rho_m} Az_L \Phi_H K$ is the fund for monetary stimulus, and $\rho_s P_0 I$ is the fund for debt bailing out. To simplify notation, we write $\Phi_H$ instead of $\Phi_H (\rho_m, \rho_b, I, A)$.
Solving this ex-post problem gives

\[ \rho_m : \pi A(z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_m} K = \]

\[ \psi \left( \pi \frac{\rho_m}{1 + \rho_m} A z_L \Phi_H K + \rho_b P_0 I \right) \left\{ \pi \frac{1}{(1 + \rho_m)^2} A z_L \Phi_H K + \pi \frac{\rho_m}{1 + \rho_m} A z_L \frac{\partial \Phi_H}{\partial \rho_m} K \right\} + \lambda_m \]

\[ \rho_b : \pi A(z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_b} K = \]

\[ \psi \left( \pi \frac{\rho_m}{1 + \rho_m} A z_L \Phi_H K + \rho_b P_0 I \right) \left\{ \pi \frac{\rho_m}{1 + \rho_m} A z_L \frac{\partial \Phi_H}{\partial \rho_b} K + P_0 I \right\} + \lambda_b \]

plus complementary slackness condition

\[ \lambda_m \rho_m = 0, \lambda_m \geq 0 \]
\[ \lambda_b \rho_b = 0, \lambda_b \geq 0 \]

the solution of this problem lies in the following four cases

1. \( \lambda_m > 0 \) and \( \lambda_b > 0 \). In this case, it is optimal for the government to do not intervene at all: \( \rho_m = \rho_b = 0 \). The above two equation implies

\[ \pi A(z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_m} K = \pi A(z_H - z_L) \frac{\partial \Phi_H}{\partial \rho_b} K = 0 \]

which cannot be true. It is optimal to engage non-zero amount of redistribution.

2. \( \lambda_m > 0 \) and \( \lambda_b = 0 \). In this case, \( \rho_m = 0 \), and \( \rho_b > 0 \). Combining the two first order condition gives

\[ \frac{\partial \Phi_H}{\partial \rho_m} = \pi \frac{A z_L \Phi_H K}{P_0 I} \]

(40)

using the definition of \( \Phi_H \) in equation (20) to compute \( \frac{\partial \Phi_H}{\partial \rho_m} \) and \( \frac{\partial \Phi_H}{\partial \rho_b} \),

\[ \frac{\partial \Phi_H}{\partial \rho_m} = \frac{A z_L}{[A z_L - (1 + \rho_m) \theta A z_H]^2} \]
\[ \frac{\partial \Phi_H}{\partial \rho_b} = \frac{1 + \rho_m}{A z_L - (1 + \rho_m) \theta A z_H} \]

equation (40) collapsed to

\[ \pi A z_L = A z_L \]

which contradicts the fact that \( \pi < 1 \).

3. \( \lambda_m = 0 \) and \( \lambda_b > 0 \). In this case it is optimal not to use debt bailout. The result for this
case is given by equation (24) and (25).

4. \( \lambda_m = 0 \) and \( \lambda_b = 0 \). In this case, \( \rho_m > 0 \), and \( \rho_b > 0 \). Recall \( \frac{\partial \Phi_H}{\partial \rho_m} \) and \( \frac{\partial \Phi_H}{\partial \rho_b} \) above, equation (38) simplifies to

\[
\frac{z_H - z_L}{z_L - \theta z_H} = \psi \left( \pi \frac{\rho_m}{1 + \rho_m} A z_L \Phi_H K + \rho_b P_0 I \right),
\]

plugging into equation equation (39) gives

\[
\frac{\pi}{z_L - \theta z_H} = \frac{1}{z_L - \theta z_H}
\]

which cannot holds as \( \pi < 1 \).

Finally, equation (26) comes from first order condition on equation (23).

### A.7 Proof of proposition 9

The first order condition with the competitive equilibrium is

\[
(1 + r) \frac{\partial \bar{V}}{\partial a} + \frac{\partial \bar{V}}{\partial d} P = 0,
\]

implying the marginal cost \( \frac{\partial \bar{V}}{\partial d} P \) equals marginal benefit. For the social planner, the problem is

\[
\max_I \left\{ \bar{V} + PI + z_h (K_h - I)^\gamma \right\},
\]

where we use capital letters to denote aggregate values. The first order condition

\[
(1 + r) \frac{\partial \bar{V}}{\partial A} + \frac{\partial \bar{V}}{\partial D} P + \frac{\partial \bar{V}}{\partial D} dP + P + \frac{dP}{dI} I - z_h (K_h - I)^{\gamma-1} = 0,
\]

as long as a non zero measure of entrepreneurs will be ex-post constrained, in which case, \( \frac{\partial \bar{V}}{\partial D} \leq -1 \), this implies

\[
(1 + r) \frac{\partial \bar{V}}{\partial A} + \frac{\partial \bar{V}}{\partial D} P = - \left( \frac{\partial \bar{V}}{\partial D} + 1 \right) \frac{dP}{dI} I \geq 0,
\]

because both \( A \) are increasing functions of \( I \). Comparing with Equation (41), the social planner’s choice of investment must be less than that of the competitive equilibrium.
A.8 A tractable industry dynamics model

This section provides a tractable industry dynamic model that explicitly shows that the externality result holds in dynamic setting. In order to analytically solve the model, we consider a simplified case when entrepreneurs ex-post productivity takes only two values $z_H$ and $z_L$, and is permanent as they enter. The probability of drawing $z_H$ is $\pi$, and $z_L$ is $1 - \pi$. Aggregate productivity $A_t$ and the short-term collateral condition $\theta_t$ can change overtime. We assume production function takes linear form. Entrepreneurs have linear utility. Similar with static model in the main text, the entrepreneurs obtains long-term debt only at entry. Long-term debt pay coupon rate $c$, and follows geometric decay with rate $\upsilon$. We denote $P_i,t$ as the price for start-up capital that is financed using long-term debt.

At date $t$, the entrepreneurs of age $j$ that does not exit the economy solves the following recursive problem

$$V_j (k_t, b_t, d_t) = \max_{\{c_t, k_{t+1}, b_{t+1}\}} c_t + \beta \left[ \lambda L_{j+1} (k_{t+1}, b_{t+1}, d_{t+1}) + (1 - \lambda) V_{j+1} (k_{t+1}, b_{t+1}, d_{t+1}) \right],$$

if the entrepreneur exits next period, she receives $L_{t+1}$, which is given by

$$L_{t+1} = A_{t+1} z_{k_{t+1}} - d_{t+1} - b_{t+1},$$

where $A_{t+1} z_{k_{t+1}}$ is the operating revenue, $d_{t+1}$ is the unpaid long-term debt that equals to $(1 - \upsilon)^{j+1} P_{i,t-j} i_{t-j}$, and $b_{t+1}$ is the short-term debt/saving. The budget constraint at date $t$ is given by

$$c_t + P_{k,t} (k_{t+1} - k_t) = A_t z_k t - c d_t + \frac{b_{t+1}}{1 + r_t} - b_t$$

where $P_{k,t}$ is the price for "used capital". From the above budget constraint, entrepreneurs finance consumption and capital expenditure with output as well as the rolling over of short-term debt, net the coupon payment of long-term debt. The borrowing constraint indicates that the entrepreneurs’ borrowing is limited

$$b_{t+1} \leq \theta_{t+1} A_{t+1} z_{k_{t+1}} - d_{t+1}$$

notice that this constraint guarantees the liquidation value $L_{j+1} \geq 0$.

Due to financial constraint, a certain fraction of capital must be left at the hands of the unproductive. The capital price $P_{k,t}$ should satisfy the following inter-temporal arbitrage condition

$$P_{k,t} = \frac{1}{1 + r} (\lambda A_{t+1} z_L + (1 - \lambda) (A_{t+1} z_L + P_{k,t+1})), \quad (43)$$

implying the unproductive entrepreneurs in equilibrium are indifferent between selling on the
market and holding for one more period. Because of a more efficient technology, the productive entrepreneurs’ valuation of used capital are strictly higher. The linear production function implies their borrowing constraint binds. We obtain the following results for the capital growth trajectory for the ex-post productive firms

\[
k_{t+1} = \begin{cases} 
\frac{(1-\theta_t)A_t z_H + P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} k_t - \frac{1-\delta}{1+r} \frac{(c-1) P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} d_t & \text{if } j \geq 1 \\
\frac{A_t z_H + P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} k_t - \frac{1-\delta}{1+r} \frac{c+1}{} & \text{if } j = 0
\end{cases}
\]

Proof. We conjecture the value function for the productive firms

\[
V(k_t, d_t, b_t) = \psi_1 k_t - \psi_2 d_t,
\]

note here the state variable \( b_t \) is dropped from the above equation because the borrowing constraint will always binds after entry so that \( b_t \) is determined by \( k_t \) and \( d_t \). Plugging into the Bellman equation and gives

\[
\psi_1 k_t - \psi_2 d_t = \beta \left\{ \lambda (1 - \theta_{t+1}) A_{t+1} z_H + (1 - \lambda) \psi_{1t+1} \right\} k_{t+1} - (1 - \lambda) \psi_{2t+1} d_{t+1}
\]

and using the budget constraint as well as the borrowing constraint on \( b_{t+1} \) yields

\[
P_{kt} (k_{t+1} - k_t) = A_t z_H k_t - cd_t + \frac{\theta_{t+1} A_{t+1} z_H k_{t+1} - d_{t+1}}{1+r} - \theta_t A_t z_H k_t + d_t
\]

this gives

\[
k_{t+1} = \frac{(1 - \theta_t) A_t z_H + P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} k_t - \frac{1-\delta}{1+r} \frac{(c-1) P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} d_t
\]

hence

\[
\psi_1 t = \beta \left\{ \lambda (1 - \theta_{t+1}) A_t z_H + (1 - \lambda) \psi_{1t+1} \right\} \frac{(1 - \theta_t) A_t z_H + P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} \tag{44}
\]

\[
\psi_2 t = \beta \left\{ \lambda (1 - \theta_{t+1}) A_t z_H + (1 - \lambda) \psi_{1t+1} \right\} \frac{1-\delta}{1+r} \frac{(c-1) P_{kt}}{P_{kt} - \theta_{t+1} A_{t+1} z_H} + \beta (1 - \lambda) (1 - \delta) \psi_{2t+1} \tag{45}
\]

For entering firms, they do not hold any short-term debt at the beginning, so the formulation is different with the incumbent firms. Denote \( V_0 (k_t, d_t) = \chi_{1t} k_t - \chi_{2t} d_t \). We have the budget constraint
\[ P_{kt} (k_{t+1} - k_t) = A_t z_H k_t - c d_t + \frac{b_{t+1}}{1 + r_t} \]

this yields

\[ k_{t+1} = \frac{A_t z_H + P_{kt}}{P_{kt} - \frac{\theta_{kt+1} z_H}{1 + r}} k_t - \frac{\frac{1 - \delta}{1 + r} + c}{P_{kt} - \frac{\theta_{kt+1} z_H}{1 + r}} d_t \]

hence

\[ \chi_{1t} = \beta \left[ \lambda (1 - \theta_t) A_t z_H + (1 - \lambda) \chi_{1t+1} \right] \frac{A_t z_H + P_{kt}}{P_{kt} - \frac{\theta_{kt+1} z_H}{1 + r}} \]

\[ \chi_{1t} = \beta \left[ \lambda (1 - \theta_t) A_t z_H + (1 - \lambda) \chi_{1t+1} \right] \frac{\frac{1 - \delta}{1 + r} + c}{P_{kt} - \frac{\theta_{kt+1} z_H}{1 + r}} + \beta (1 - \lambda) (1 - \delta) \chi_{2t+1} \] (47)

In what follows, we focus on the steady state of the model, in which aggregate technology \( A_t \) and \( \theta_t \) takes constant values, denoted as \( A \) and \( \theta \). Due to the linear setting, we have

\[ P_k = \frac{A z_L}{r + \lambda} \] (48)

\[ \psi_1 = \frac{\beta \lambda (1 - \theta) A z_H \zeta}{1 - \beta (1 - \lambda) \zeta} \] (49)

\[ \chi_1 = \psi_1 \] (50)

\[ \zeta = \left( (1 - \theta) z_H + \frac{z_L}{r + \lambda} \right) / \left( \frac{z_L}{r + \lambda} - \frac{\theta z_H}{1 + r} \right) \]. The first equation is from equation (43); the second equation is from equation (45); and the third equation is from equation (47). With the three values above, \( \psi_2 \) and \( \chi_1 \) can be solved correspondingly from equation (45) and (47).

**The Ex-ante Problem.** Ex-ante, entrepreneurs investment is chosen by solving the following program

\[
\max_i \left\{ \lambda \left[ (1 - \pi) z_L k + \pi z_H k - P_i i \right] + (1 - \lambda) \left[ (1 - \pi) \left( (P_k + z_L) k - P_i i \right) + \pi \left( \chi_1 k - \chi_2 P_i i \right) \right] \right\}
\]

where \( k \equiv k_e + i \) is the start-up capital. This yields

\[ P_i^* = \frac{\lambda \left[ (1 - \pi) z_L + \pi z_H \right] + (1 - \lambda) \left[ (1 - \pi) \left( P_k + z_L \right) + \pi \chi_1 \right]}{1 + \pi (1 - \lambda) (\chi_2 - 1)}, \]

where \( P_k \), \( \chi_1 \) and \( \chi_2 \) are given by equation (49), (50) and (50). The social planner takes into account that its decision on \( I \) affect capital price, in which

\[ P_i^{**} = \frac{\lambda \left[ (1 - \pi) z_L + \pi z_H \right] + (1 - \lambda) \left[ (1 - \pi) \left( P_k + z_L \right) + \pi \chi_1 \right]}{1 + \pi (1 - \lambda) (\chi_2 - 1) \left( 1 + (1 - \gamma) \frac{I^{**}}{K_h - I^{**}} \right)} \]
in both allocations, the amount of investment $I^*$ and $I^{**}$ are given by capital supply function

$$K_h - I = \left( \frac{P_i}{z_h} \right)^{\frac{1}{\gamma}}.$$ 

Note here we need $z_h \geq 1 - \lambda (1 - \theta)$ so that the productive do not absorb all the capital in steady state so that economy reaches first best. In other words, $\theta$ cannot be too large. The intuition here follows the discussion in Assumption (3). Clearly, the over-borrowing results holds in this dynamic setting as $P^{**}_i < P^*_i$.

### B Numerical Steps

In this section, I describe the numerical algorithm implemented to solve the dynamic model. Given the set of model parameters to be calibrated, we solve the firm’s value function and the corresponding policy function using the standard value function iteration (VFI) method. Our assumption that firm knows its next period productivity draw simplifies the model solution by reducing the model dimension by one. Therefore, entrepreneur’s state variable contains 4 dimensions: net worth ($a$), idiosyncratic productivity ($z$), age ($j$), as well as the long-term debt holding ($d$). The net worth state is discetized into 100 (log) equal spaced grids from 0 to $a_{\text{max}}$; the idiosyncratic productivity is discritized using Tauchen (1986) method using 9 grid point for $\pm 3$ standard deviations; age can only take discrete values from 0 to 100; the state for initial long-term debt is discritized from $d_{\text{min}}$ to $d_{\text{max}}$ using 20 equally spaced grid points. For grids on net worth and age, I check that further increasing the number of grid point does not change aggregate moments. The total number of discritized grids are: $100 \times 9 \times 101 \times 20 = 1,818,000$. Given this large state space, the search for optimal policy is programmed to run in parallel for each grid point ($a, z, j, d$) and the computation is done in a HKUST GPU server with 2× Intel E5-2660 v2 together with 2×Nvidia Tesla K40, using the OpenACC Complier\(^{14}\). For each state, we only need to search on one dimension. Using the standard Golden Section Search method as well as a linear interpolation for the off-grid choices enables me to find the optimal policy in about 10 minutes for each parameter set.

**Solving the Steady State Equilibrium** In the model, firm’s long-term debt holding at entry, $d$ affect their future value function through debt overhang and coupon payment. In the competitive equilibrium, entrepreneur’s demand for investment is conditional on the capital price $P_i$. The following steps solve for the decentralized $P_i$: (1) guess $P_i$. Conditional on this $P_i$, choose firm’s investment decision by solving the program in equation

$$\max_i \{ E[V(a_t, d_t, z_t, 0)] \}$$

where $d = P_i i$ and (2) update $P_i$ using bisection search method until the capital market clears.

\(^{14}\)There are several advantages using this Complier: (1) it automatically schedule the thread; (2) it optimizes the data locality; (3) it provides different level of parallelism;
The solution for constrained efficiency is similar, except that when choosing $I$, the social planner takes into account the positive effect of $I$ on $P_i$. After choosing for $I$, the long-term debt decision on every entering firms are solved.

**Solving the Transition Dynamics** We now describe the solution procedure for solving the transition dynamics for both the decentralized economy and also for the constrained efficient case. We start by describing the competitive equilibrium. Our solution procedure adopt the standard “shooting” method (Guerrieri and Lorenzoni, 2017). Essentially, what we want to solve is the fixed-point of the capital price sequence on $(P_{it}, I_t)$ for each $t$ along the transition path satisfies: (1) the entrepreneurs’ first order condition; (2) the capital supply equation (1b).

Given this sequence of allocations, aggregation of the rational agents’ optimal decisions, implies the same price vector along the transition path. Iterative method is used to find the fixed-point price trajectory. The exact solution method consists 5 main modules:

1. We guess the price for investment $P_{0it}$ along the transition path, where $1 \leq t \leq T$. $T$ is a sufficiently large number where we assume transition completes. We tried various forms of initial price trajectory $P_{0it}$ to make sure our results is not sensitive to this initial specification. For $T$, we initially set it at 150, and numerically verify an increase in $T$ does not affect our results.

2. We iterate backwards to get optimal decisions along the whole transition path. Specifically, because we have value function for post-transition steady state, i.e. after period $T+1$, we can obtain the optimal decisions for entrepreneurs’ backwards for period $T$, $T-1$, ..., 2, 1, by solving dynamic programming equation backwards for each type of agents. In our quantitative model, all future price vectors $P_{0it}$ will affect current price due to the presence of long-term debt.

3. Starting from period 1, we simulate the economy forward using the optimal decisions obtained in step 2. The initial distribution is simply taken from the steady state before aggregate shock. In this step, we have a whole set of distribution dynamics along the transition path.

4. Given that we know exactly how distribution changes over time (from step 3), and optimal policies (from step 2). We compute, period by period, for price vectors that clears capital market $P_{it}$.

---

\[15\] Consider time $t + j$, where $j \in [0, T]$, then $P_{0it}$ with $t \geq j + 1$ will enter into the value function of entering entrepreneurs at date $t + j$ because $P_{0it}$ affect the capital holding of future entering firms. It in turn affect the capital rental rate, and thus affect the current capital price. Because of the introduction of a long-term debt here, we get this additional complication in comparing (Guerrieri and Lorenzoni, 2017).
5. We update the price vector by using the following relaxation algorithm

\[ P^{k+1}_{it} = \Delta \times P^k_{it} + (1 - \Delta) \times P^{UPD}_{it} \]  

(51)

where \( \Delta \) is a relaxation parameters that controls the speed of convergence. We choose \( \Delta = 0.2^{16} \), and we loop until

\[ ||P^{k+1}_{it} - P^k_{it}|| < 0.0001. \]  

(52)

To illustrate, consider at date \( t \), a one time, AR(1) shock to the collateral value \( \theta \). To solve for the constrained efficient case, we choose the whole sequence of price vector in order to maximize the total welfare in the transition path. We set the initial value of price vectors to the decentralized case, and we use the standard derivative free “Nelder-Mead” algorithm to find the optima. This is for fast convergence. As a robustness check, we also let the initial value to be the steady state price through the whole transition path. The two method converges to identical results.

How we solve for decentralized allocation and the planner allocation in transition dynamics. For date \( t + j \) during transition, the entering entrepreneurs solves

\[
\max_{\{I_{t+j}\}} \{ \mathbb{E} [V (a_{t+j}, d_{t+j}, z_{t+j}, 0) | \theta_{t+j}] \}
\]

such that

\[
a_{t+j} = k_e + i_{t+j},
\]

\[
d_{t+j} = P_{i,t+j}i_{t+j},
\]

as well as the market clearing condition

\[ P_{i,t+j} = \gamma (K_h - I_{t+j})^{\gamma-1}. \]

We have the social planner’s program during transition dynamics is given by

\[
\max_{\{I_{t+j}\}} \left\{ \sum_{j=0}^{\infty} \mathbb{E} [V (A_{t+j}, D_{t+j}, z_{t+j}, 0) | \theta_{t+j}] \right\},
\]

\(^{16}\)One can increase this number to make the iteration faster. However, practically this will also result in non-convergence of the price vector.
where

\[ A_{t+j} = K_e + I_{t+j} \]
\[ D_{t+j} = \gamma (K_h - I_{t+j})^{\gamma-1} I_{t+j} \]

and the social planner can solve this program period by period, i.e. do maximization for each \( j \). This is due to the setting that risk-free rate and wage rate are fixed, so that different cohort do not spill over to each other. Otherwise, the model should inherit the property of a standard general equilibrium perpetual youth model, in which the capital decision of one generation induce spill-over across other generations through capital return. The choice of \( I_{t+j} \) yields an “implied asset price” \( P_{t+j} = \gamma (K_h - I_{t+j})^{\gamma-1} \).

C Data Appendix

To maximize data coverage, we append WRDS-ORBIS (Wharton Research Data Services), Historical-ORBIS, and WRDS-Amadeus dataset. The WRDS-ORBIS dataset is a relatively new dataset that compiles Bureau Van Dijk’s historical file. As the product has been introduced only in 2008. The realized results have relatively low coverage before 2008. The ORBIS-AMADEUS dataset is also directly downloaded from the Wharton Research Data Services. Following Gopinath et al. (2017), we combine their unique Bureau Van Dijk Index. If the index appears in both datasets, we keep the one from the ORBIS historical file for a longer span. For our purpose, the ORBIS (WRDS or Historical) have several advantages. First, it allows us to make comparison through time, using financials older than 10 years, which is the maximum possible for their online version. Second, the product makes sure BvD ID number are compatible with the corresponding version of ORBIS so that one do not need to worry about the ID change overtime. Third, Bureau Van Dijk has checked the link of historical data to all firms, which allows us to clearly identify if a firm enters/exiting from operation. Fourth, the industry classification has been made consistent across different vintages. Its most detailed classification: 4 digit NACE (Nomenclature générale des Activités économiques dans les Communautés Européennes)\(^\text{17}\), has around 230 sub-sects within the manufacturing industry, with a comparable detailedness to 6 digit NAICS (North American Industry Classification Codes), which have around 270 sub manufacturing sectors. We keep dataset from 1999-2015, NACE Rev-2 code from 1000 to 3399 (manufacturing), and we only keep unconsolidated accounts. Finally, we obtain dataset that covers around 3,300,000 firms, with 1/3 being manufacturing.

\(^{17}\)Statistical classification of economic activities in the European Communities
Table 4 shows firm coverage at the year 2006 for our final cleaned data.

<table>
<thead>
<tr>
<th>iso2</th>
<th>size</th>
<th>employment</th>
<th>staff</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>1-19 employee</td>
<td>0.007945</td>
<td>0.005780</td>
<td>0.01486</td>
</tr>
<tr>
<td>DE</td>
<td>20-249 employees</td>
<td>0.3517</td>
<td>0.3167</td>
<td>0.3329</td>
</tr>
<tr>
<td>DE</td>
<td>250+ employees</td>
<td>0.6404</td>
<td>0.6775</td>
<td>0.6523</td>
</tr>
<tr>
<td>ES</td>
<td>1-19 employee</td>
<td>0.3350</td>
<td>0.3037</td>
<td>0.2738</td>
</tr>
<tr>
<td>ES</td>
<td>20-249 employees</td>
<td>0.6102</td>
<td>0.6310</td>
<td>0.6485</td>
</tr>
<tr>
<td>ES</td>
<td>250+ employees</td>
<td>0.05480</td>
<td>0.06524</td>
<td>0.07762</td>
</tr>
<tr>
<td>FR</td>
<td>1-19 employee</td>
<td>0.1151</td>
<td>0.1232</td>
<td>0.06733</td>
</tr>
<tr>
<td>FR</td>
<td>20-249 employees</td>
<td>0.2483</td>
<td>0.2066</td>
<td>0.1851</td>
</tr>
<tr>
<td>FR</td>
<td>250+ employees</td>
<td>0.6366</td>
<td>0.6702</td>
<td>0.7476</td>
</tr>
<tr>
<td>IT</td>
<td>1-19 employee</td>
<td>0.1640</td>
<td>0.1466</td>
<td>0.1700</td>
</tr>
<tr>
<td>IT</td>
<td>20-249 employees</td>
<td>0.6580</td>
<td>0.6545</td>
<td>0.6403</td>
</tr>
<tr>
<td>IT</td>
<td>250+ employees</td>
<td>0.1780</td>
<td>0.1988</td>
<td>0.1897</td>
</tr>
<tr>
<td>NO</td>
<td>1-19 employee</td>
<td>0.1706</td>
<td>0.3760</td>
<td>0.6304</td>
</tr>
<tr>
<td>NO</td>
<td>20-249 employees</td>
<td>0.2545</td>
<td>0.1149</td>
<td>0.07595</td>
</tr>
<tr>
<td>NO</td>
<td>250+ employees</td>
<td>0.5748</td>
<td>0.5091</td>
<td>0.2937</td>
</tr>
<tr>
<td>PT</td>
<td>1-19 employee</td>
<td>0.2771</td>
<td>0.2231</td>
<td>0.1973</td>
</tr>
<tr>
<td>PT</td>
<td>20-249 employees</td>
<td>0.6138</td>
<td>0.6336</td>
<td>0.6433</td>
</tr>
<tr>
<td>PT</td>
<td>250+ employees</td>
<td>0.1092</td>
<td>0.1433</td>
<td>0.1594</td>
</tr>
</tbody>
</table>

We proceed to productivity estimation. Following the standard practice, we deflate firm nominal sales, value added, wage bills, using an output price deflator. Since firm-level price is not observed in our dataset, we use the gross output price deflators from the EUROSTAT at the two-digit industry level. We measure the capital stock as the value with the price of investment goods. We use country specific price of investment from the World Development Indicators. Fixed assets include both tangible and intangible fixed assets. Our results change very little when we exclude firm intangibles. To construct sector level variables (MPK dispersion, loan growth, etc), we drop any country-sector-year pair with less than 2 firms that report non negative values for value added, employment, material cost, as well as cost of employee to obtain an meaningful sectoral level aggregation result.

To estimate firm-level productivity, we follow the Woodridge adaption of the standard Leveshon and Petrin method. The estimation is by two digit NACE pairs and we conduct the production function estimation separately for each country. This allows for the possibility of potentially different capital and labor share for different countries. The following table reports

\(^{18}\)Due to the disclosure policy on this dataset, we only replot the data moments that are identical with Gopinath et al. (2017)
Table 5: Summary Statistics on $\alpha + \gamma$

<table>
<thead>
<tr>
<th>iso2</th>
<th>meansh</th>
<th>mediansh</th>
<th>maxsh</th>
<th>minsh</th>
<th>sdsh</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>0.7890</td>
<td>0.8033</td>
<td>0.9065</td>
<td>0.6241</td>
<td>0.07640</td>
</tr>
<tr>
<td>ES</td>
<td>0.7807</td>
<td>0.7763</td>
<td>0.8548</td>
<td>0.7004</td>
<td>0.03447</td>
</tr>
<tr>
<td>FR</td>
<td>0.7716</td>
<td>0.7793</td>
<td>0.8917</td>
<td>0.5407</td>
<td>0.07379</td>
</tr>
<tr>
<td>IT</td>
<td>0.6700</td>
<td>0.6668</td>
<td>0.7613</td>
<td>0.5827</td>
<td>0.04215</td>
</tr>
<tr>
<td>NO</td>
<td>0.7388</td>
<td>0.7438</td>
<td>0.8914</td>
<td>0.5336</td>
<td>0.1418</td>
</tr>
<tr>
<td>PT</td>
<td>0.7528</td>
<td>0.7462</td>
<td>0.8608</td>
<td>0.6304</td>
<td>0.05515</td>
</tr>
</tbody>
</table>

This table reports the summary statistics for estimated elasticities $\alpha + \gamma$ for each country.

the estimated capital and. Our estimation gives a median markup value of around 0.25, as is shown in Table 5. To construct the firm-level productivity shock for the regression analysis, we follow Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) to run the following regression

$$\log(A_{jt}) = \rho \log(A_{j,t-1}) + i_j + \lambda_{ct} + \varepsilon_{jt},$$

(53)

in which we regress the current period log productivity for firm $j$ on its lagged value, controlling for firm and country × year fixed effect. The residual $\varepsilon_{jt}$ is thus the productivity shock.